HAND WRITTEN NOTES:

OF

CIVIL ENGINEERING

SUBJECT:

DESIGN OF STEEL STRUCTURE
Design of Steel Structures

Riveted connection
Welded connection
Eccentric connection
Tension member
Compression members
Beams
Plate girder
Industrial building
Plastic Analysis - Most Imp

By - Rekha Thakur
Riveted Connection

Rivets are generally made of mild steel, and for normal construction purposes, Fe 410S grade of steel. 410 N/mm² ultimate strength of steel. This grade is suitable for rivetting for all thicknesses, but for welding, it is suitable only up to 20 mm thickness of member.

For thicknesses more than 20 mm welding, grade of steel is used. For member, they are Fe 410 W4.

The other grades of steel are Fe 570 HT. HT = high tension.

High tension grade of steel has higher strength, and they are more resistant to atmospheric corrosion, hence this grade is adopted when reduction in dead wt is desired and when structure is prone to aim corrosion.

Sp. gravity = 7.85

Density = 7850 kg/m³

Unit wt = 78.5 KN/m³

\[ E = 2 - 215 \times 10^5 \text{ N/mm}^2 \]

\[ \alpha = 1.2 \times 10^{-6} ^{\circ}C \]
Advantage of steel structure over aluminium str.

Strength per unit wt. of aluminium is much greater than that of steel.

However, $E_{al} = \frac{1}{3} E_{st}$

Hence Al structure is more prone to buckling. To overcome this, the cross-sectional area of Al str. needs to be increased. Even then a saving of 50% in wt. results by the use of Al str.

However, Al is almost 8 times costlier than steel. Thus greater economy is achieved by the use of steel str.

$\kappa_{al} = 2 \kappa_{st}$

Hence it can not be encased in concrete as steel is normally done.

Note: Encasing is normally done for aesthetic & fire resistance.

Maintenance cost of Al structure is less as it is not prone to corrosion.

Method of Design:

Elastic method of design is used.

Simplified mild steel stress-strain curve

The structure is designed in such a way that stress in the member do not exceed permissible stress.
To design the structure we need to access the loading to which the structure will be subjected to, hence there is uncertainty in loading. Similarly there is uncertainty in material property.

Also to simplify the analysis certain assumptions are made because of which error will be introduced in the result, to account for all then we use F.O.S.

\[
\text{F.O.S. for tension} = 1.67 = \frac{1}{0.6} \quad f_s = 0.6f_y
\]

\[
\text{F.O.S. for bending} = 1.5 = \frac{1}{0.66} \quad f_s = 0.66f_y
\]

Note: In case of bending there is a margin to resist additional loading beyond the point of first yield. However in tension there is no margin beyond the point of first yield. Hence FES in tension is more as compared to bending.

**Nominal Dia of rivet**

It is the dia of shank of rivet in cold condition.

Cold and Hot rivetting.

Cold and hot rivetttings are methods of rivetting. Cold rivetting is not adopted for dia > 10 mm.

In hot rivetting rivets are heated to 550 - 1000 °C. They are inserted in the hole made in the member and hammering is done so as to make head on the hammer side. The diameter of hole in the member should be more than the nominal dia of rivet so that the rivets can be easily inserted in heated condition.
In comp. gross area is effective and in tension net area is effective in resisting loads.

Rivets in group subjected to direct load share the load equally (if they are of the same diameter).

Actually stress is assumed to be equally shared.

\[
\text{Force taken by one rivet } = \frac{P}{g}
\]

The outer rivet will be stressed more as compared to the inner rivet hence in our line we do not adopt more than 5 rivets otherwise buttressing effect will occur and rivets will start breaking one by one.

Bending stress in rivet is neglected under normal situation but if grip length is more the bending can not be neglected in that case additional precautions are taken as follows. If punching is of larger depth, additional rivets are provided on punching extension. This additional rivets absorbs the effect of (reduce the effect of) bending stress.

Recommendation: If the grip of rivet exceeds 6 times the dia of rivet hole, no of rivets required by normal calculation should be increased by not less than 12 for each additional grip of 1/16 in.
provided on backing extension.

Dia of hole = 20 mm
Grip length = 75 mm
Load = 100 KN

Strength of 1 rivet = 20 KN

Calculate the no. of rivets required

6 x dia of hole = 6 x 20 = 120 mm
grip over 120 mm = 152 - 120 = 32 mm

So additional no. of rivets (over and above from normal calc.)

= 1% of the no. of rivet obtained from normal calculation per 1.6 mm

no. of rivet req. by normal cal. = \( \frac{100}{20} = 5 \)

so add. no. of rivet required = \( \frac{3.20^2 + 1 X 5}{1.6} \)

total no. of rivet = 5 + 1 = 6

Rivet fills the hole completely, i.e., to calculate strength of rivet we use the gross dia of rivet, i.e., the hole dia.

Pitch & Gauge

\[ \text{gauge length} \]

Distance b/w centre line of two rivets in the dia of force is called pitch of this b/w centre of two rivets at right angle to the dia of force is called gauge.
Dia of hole  =  Nominal dia of rivet + 1.5 mm for dia ≤ 25 mm
Dia of hole  =  Nominal dia of rivet + 2 mm for dia > 25 mm
Dia of hole is also called as gross dia of rivet. (This is under the assumption that rivet fills the hole completely)

Unwin’s formula

Dia of rivet to suit the thickness of member

\[ d_{mm} = 6.05 \sqrt{t_{mm}} \]

(Not recommended by IS code)

It is a guideline.

It is the thickness of thinner member being joined.

Normally adopted dia of rivets are 10, 12, 16, 18, 20, 22 in (mm) 15, 20, 25 mm.

Assumptions in riveted connection.

Friction b/w the plates is neglected.

In hot rivetting because of gripping friction develops b/w the plates. So long as the external force overcomes the frictional force, rivet will not be subjected to any stress once the friction is overcome, the rivet will deform and hence will start sharing loads but it is difficult to quantify the amount of friction hence it is neglected and all the forces are assumed to be resisted by rivet only.

Shear stress is uniform over the cross section of rivet.

Actual variation of shear stress

Assumed shear stress
Max. S.F. resisted by a rivet: \[ \sigma_{rm} \leq \frac{\sigma_{tm} \times T d^2_{hole}}{4} \]

**Note:** Shear failure is a sudden failure hence for shear it is large.

Above assumption is a simplifying assumption.

Distribution of direct stress on the portion of plate by the rivet hole is uniform.

![Diagram of rivet](image)

Actual stress variation

Assumed stress variation

- \(B\) : Thickness of plate
- \(d'\) : Dia. of hole
- \(\sigma_t\) : Per. tension stress

\[ P_{max} = (B - 3d') \times t \times \sigma_t \]

**Note:** Because of stress concentration, stress is normally 2-3 times that of \(\sigma_t\).
Transmission of load through rivets

\[ P = \pi d'^2 t \]

- Bearing stress
- Shearing stress

\[ \sigma_{br} = \frac{P}{A} \]

Hole dia. = \( d' \)

Max bearing force resisted by top portion of rivet

- By top \( (d'^2 t_1) \sigma_{br} \)
- By bottom \( (d'^2 t_2) \sigma_{br} \)

Max force resisted by rivet in bearing = \( d'^2 t \sigma_{br} L \)

L = min thickness

Max force resisted by rivet in shearing = \( \frac{4d'^2 t}{\pi} \sigma_s \)

\( \sigma_s \) = permissible shearing stress

The strength of rivet is min of shearing and bearing strength. This is called rivet value of the rivet. 

\[ \sigma_b \]

\( \sigma_b \) = strength of rivet

If the rivet fails in shear by shearing on one plane, the rivet is said to be in single shear.
12

$d' = $ Dia of hole

Bearing strength of rivet \( = d' \times t \times \sigma_{bt} \)

\[ t = \min \left( t_1, \left( t_2 + t_3 \right) \right) \]

Shearing strength of rivet \( = 2 \frac{d'^2 t}{4} \times \sigma_s \)

In this case rivet is said to be in double shear.

Note: Max. strength of rivet is taken as that corresponding to double shear.

Type of Joints

Lap joint

One pitch, length

Single Rivetted Lap Joint
Double rivetted lap joint

Single rivetted single cover butt joint

Eccentricity is eliminated in the case of double cover butt joint.

\[ m = \frac{pe}{2} - \frac{pe}{2} = 0 \]
Failure of Rivet Joint

1) By tearing of plate b/w rivet hole and edge of plate

2) By tearing of plate b/w rivet

It is a type of shearing failure of plate at ends and occurs due to insufficient edge distance. It is prevented by keeping the edge distance to be twice the diameter of rivets hole.

\[ d' - \text{dia of hole} \]
\[ t - \text{thickness of plate} \]

\[ \text{N.E.A in tension} = (8 - 4d')t \]

Strength of main plate in tearing = \( (8 - 4d')t \delta_{at} \)

\( \delta_{at} \) - permissible stress in axial tension

\[ \text{Strength of main plate/pitch length} = (b - d')t \delta_{at} \]
In case of double rivetted butt joint strength of joint for pitch length will be

\[ (b-d) t \delta \alpha \]

Shearing failure of joint by shearing of rivets
Strength of joint in shearing is equal to summation of shearing strength of all rivets in the joint

\[ \text{strength of joint in shearing} = \frac{\pi d^2}{4} \times \delta \times 1 \]

Rivets are in single shear No. of rivet/pitch length = 3
Strength of joint in shearing per pitch length

\[ = \frac{\pi}{4} \left( d^2 \right) \delta \times 1 \]

Strength of triple rivetted double corner butt joint in shearing per pitch length

\[ = \frac{\pi}{4} \left( d^2 \right) \delta \times 3 \]

St. of one delt in double shear
Bearing or crushing of rivet

![Diagram]

Strength of joint per pitch length in bearing:

\[ \sigma_b = \frac{d'}{t} \times \sigma_{br} \times L \]

where:
- \( d' \) = Min thickness of plate
- \( \sigma_{br} \) = Bearing failure of plate
- \( L \) = Combined thickness of two cover plate or thickness of main plate whichever is min

![Diagram]

Bearing failure of rivet
Bearing failure of plate

Strength of joint is the minimum of shearing, bearing and tearing strength of joint.

Efficiency of Joint

\[ \eta = \frac{\text{Strength of joint (Min of shearing, bearing, tearing)}}{\text{Strength of main plate without deduction for hole}} \]

Note: For efficient utilization of material, rivets and plates should fail simultaneously.

IS Code Recommendation.

1) Permissible stress in rivets

<table>
<thead>
<tr>
<th>Type</th>
<th>Axial tension</th>
<th>Shearing</th>
<th>Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shop rivet</td>
<td>100 MPa</td>
<td>100 MPa</td>
<td>300 MPa</td>
</tr>
<tr>
<td>Power-driven</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>250 MPa</td>
</tr>
<tr>
<td>Hand-driven</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>250 MPa</td>
</tr>
</tbody>
</table>

For field rivetting, the above permissible values are reduced by 10%
Permissible stress values are increased by 25% in case of wind or earthquake loading.

Note: Wind and/or EQ forces act for very short duration for short duration members force will increase and hence the no. of rivet requirement will increase significantly if permissible stress are kept constant but over there is a margin between permissible stress and yield stress, and if that margin is utilized for that short duration no. of rivet requirement will not increase significantly hence economy will result thus for that short duration FOS is reduced and hence permissible stress is increased.

Min Pitch:

Min pitch = 2.5 times the nominal dia of rivet

Max Pitch:

a) Dis b/w. the centres of two consecutive rivets in the dia. of stress should not exceed 16t or 250 mm whichever is less in tension member where t is the thickness of thinner flange

b) In comp. it is

12t or 200 mm whichever is less
for tacking sheets & max pitch is 32 t or 300 mm which ever is less.

Tacking rivets are used to ensure that the two members being joined along the length behave as a single unit.

packing

Rivets on packing extension

If the difference of thickness of two plates being joined \((t_1 - t_2)\) is greater than 6 mm, then additional rivets shall be provided on packing extension.

No. of additional rivets will be 2.5% of actual no. of rivet obtained from normal calculation per 2 mm thickness of packing.

Arrangement of rivets.
Force in the main plate before 1-1 is less than that before 1-1 because rivets at section 1-1 absorb some force and transmit it to the cover plate. The same thing holds for 2-2 and 3-3, therefore main plate required max. net area at section 1-1 than at other sections. Thus it is desirable to have minimum no. of rivets at section 1-1 providing max. N.E.A. No. of rivets can be increased in inner radius that is 2-2 & 3-3. Thus diamond riveting is better as compare to chain riveting.

Tearing strength of Diamond

\[
\begin{align*}
1-1: & \quad (B-d') t \sigma_{at} + \sigma_{at} = P \\
2-2: & \quad (B-2d') t \sigma_{at} + Rv = P \\
3-3: & \quad (B-3d') t \sigma_{at} + 3Rv = P
\end{align*}
\]

\[
\begin{align*}
8-2d & \quad (B-2d) t \sigma_{at} + 2Rv = P \\
8-3d & \quad (B-3d) t \sigma_{at} + 4Rv = P
\end{align*}
\]
If
\[ R_v - d'^2 t \sigma_{at} \leq 0 \]

section 2-2 will fail in tearing

2-2 more critical as compared to 1-1

At

for chain riveting most critical section is section 1-1. If plate does not fail at section 1-1, it will never fail at section 1-2.

4 3-3.

for cover plate section 3-3 is most critical.

A member of a truss consists of 2 angles 6 mm thick placed back to back. It carries a direct load of 100 KN and is connected to a gusset plate 8 mm thick. Determine the no. of power-driven field rivets of 18 mm dia required for the joint:

\[ d' = 18 + 1.5 = 19.5 \]

Strength of rivet in shearing:
\[ \frac{2 \times d'^2}{t} \times 90 = \]
\[ = \frac{2 \times 19.5^2}{90} = 53.757 \text{ KN} \]

in bearing:
\[ 19.5 \times 8 \times 90 \times 300 \times 9 = 4212 \text{ KN} \]

Riveting value \( R_v = 53.757 \text{ KN} \)

No. of rivets:
\[ \frac{100}{4212} = \frac{2 \times \text{rivet}}{\text{rived}} = \frac{3 \times \text{rivets}}{\text{rivets}} \]

\[ 170 \text{ mm} \]

45 mm
Note
Shearing strength of rivet $\alpha \cdot d^2$

Bearing strength of rivet $\alpha \cdot d$

Hence when shearing governs the design, it is better to use small no. of large dia rivets. If however, bearing governs the design, it is better to use large no. of small dia rivets.

Q. The plate of a boiler 6 mm thick is connected by single rivetted lap joint with 16 mm dia. haz. driven shop driven at 50 mm pitch. Calculate the efficiency of joint.

$\bar{d}^1 = 16 + 1.5 = 17.5$

$t = 50 \times 6 \times 150 = 45$ KN

Tearing strength of plate = $(50 - 17.5) \times 6 \times 150 = 29.25$ KN

Strength of rivet in shearing = $\frac{\pi \cdot d^2}{4} \times 100 = 24.053$ KN

Bearing = $17.5 \times 6 \times 270 = 28.95$ KN

$N = 24.053 + 28.95 = 53.053$ KN

$\eta = \frac{24.053 \times 100}{45} = 53.45\%$
A tie member has to translate a pull of 400 tons. Design a butt joint to connect two no. of 12-mm thick plate and also find its efficiency for power driven shop rivet.

No. of rivets and its dia
Arrangement of rivets
Width of main and cover plate
Thickness of cover plate
Efficiency

Dia of rivet = \( 6.05 \sqrt{12} = 20.95 \text{ mm} \)

Let us adopt 20 mm dia rivet. \( \Rightarrow d' = 20 + 1.5 = 21.5 \text{ mm} \)

Rivet value = \( \text{Min of (Shearing strength, Bearing strength)} \)

Shearing strength = \( 2 \times \frac{7}{4} \times (2.5)^2 \times 100 = 72.61 \text{ KN} \)

Bearing strength = \( \)

Assuming that combined thickness of two cover plate is more than main plate

Bearing strength = \( d' \times t \times 300 = 77.41 \text{ KN} \)

Rivet value = 72.61 KN

No. of rivets = \( \frac{900}{72.61} = 5.5 \)

Adopt 6 no. of 20 mm dia rivet

Arrangement of rivets will decide the dimensions of main & cover plate.

Let us adopt diamond riveting pattern.
Width of main plate will be provided such that the main plate does not tear off at any section 1.2.1.3

Tearing strength of main plate at section 1-1

\[(B - d')\times 12 \times 150 + h_v > 400 \times 10^3\]

\[B - d' > 222.22\]

\[B > 222.22 + 21.5\]

\[B > 243.72 \text{ mm}\]

Tearing strength at 2-2

\[(B - 2d')\times 12 \times 150 + h_v > 400 \times 10^3\]

\[B > 224.88 \text{ mm}\]

at 3-3

\[(B - 3d')\times 12 \times 150 + 3R_v > 400 \times 10^3\]

\[B > 165.7 \text{ mm}\]

Let us adopt width of main plate = 250 mm

Adopting the width of cover plate same as that of main plate

Tearing strength of cover plate at section 3-3

\[(250 - 3d')\times 2t \sigma_{at} > 400\]
t > 7.081 - 7.187 mm

Recommendation - Min thickness of plate in steel structure should be adopted 6 mm when it is not exposed to atm. and 8 mm if it is exposed to atm.

Let us adopt $t = 8\text{mm}$  

Arrangement — Diamond
No. of rivet — 6
Width of main plate — 250 mm
Width of cover “ — 250 mm
Thickness of cover plate — 8 mm

$C = 2 \times 21.5 = 43$

$f_m = 2.5 \times 20 = 50$

$e = 4 \times 21.5 + 5 \text{mm} = 4 \times 43 + 4 \times 50 + 5 = 377 \text{mm}$

$e = 4 \times 21.5 + 5 \text{mm} = 4 \times 43 + 4 \times 50 + 5 = 377 \text{mm}$

Efficiency = \[
\frac{\text{Min of Shearing, Bearing, Tearing St. of Joint}}{\text{Strength of main plate without deduction for hole}}
\]

Min (Shearing St. of Joint) = $6\text{KN} = 6 \times 72 - 61$

Bearing St. of Joint = 435.66 KN

Tearing strength of main plate = $250 - 21.5 \times 12 \times 150 = 411.3 \text{KN}$

Cover plate = $250 - 3 \times 21.5 \times 16 \times 150 = 445.2 \text{KN}$
hence strength of joint = 411.3 kN

\[
\eta = \frac{411.3 \times 10^3 \times 100}{250 \times 12 \times 150} = 91.4 \%
\]

2. Design a rivetted splice for a tie of a steel bridge

20 cm wide 20mm thick carrying an axial load if 500 kN.

Use 12mm thick cover plate and 22 mm dia rivet

\[\sigma_{stl} = 150 \text{ MPa}, \quad \sigma = \text{permissible stress in shear} = 100 \text{ MPa} \]

\[\sigma_{tr} = 300 \text{ MPa} \]

SOL.

\[d'_1 = 22 + 7.5 = 29.5 \text{ m} \]

Shearing

rivet value = \[\min \left( \frac{2 \times 7 \times d'_1^2 \times 100}{4}, \quad d' \times 20 \times 300 \right) \]

= \[2 \times 93.37 = 141 \text{ kN} \]

rivet va = 86.74 kN

No. of rivet = \[\frac{500}{86.74} = 5.76 \]

Adopt 6 rivet of 22 mm dia

Arrangement

leaving strength of section 1-1 = \[(B-d') x 20 x 200 x 150 \]

= 529.5 kN = 1500 safe

leaving strength of section 2-2 = \[(B-2d') x 20 x 150 + 86.74 \]

1.1
$3 - 3 = (8 - 3d') \times 20 \times 150 + 3 \times 86.74$

$= 648.72 \text{ KN}$

For cover plate (Adopting width of cover plate = width of main plate)

$= (200 - 3d') \times 24 \times 150$

$= 466.2 \text{ KN} \leq 500 \text{ KN}$

So cover plate is not safe in tension.

After adopting diamond arrangement connection will

\[ \text{see } 3-3 \]

$55 \quad 55 \times 2 + 47 \times 2 = 29.4$

$2x = 3.5 \Rightarrow 2x = 47$

So take

$45$

$55$

$55$

$45$

So 3 rivets can be accommodated

Hence we can apply 3 rivet at seen 3-3 by 100 cover plate.

Let us adopt chain rivetting.

Check tensile strength of main plate at seen 1-1

\[(8 - 2d') \times 20 \times 150 = 454 \text{ KN} < 500 \text{ KN} \]

Not safe
Hence let us adopt

(i) or (ii)
determine the no. and pattern of 20 mm dia rivet used for connection

permissible stress in shearing in rivet = 102.5 N/mm²

\[ \sigma_{br} = 236 \text{ N/mm}^2 \]

for connection of angle I + II rivet is in single shear but for connection of angle III with gusset rivet is in double shear.

Hence rivet value for I, II = \( \min \) (shearing strength, Bearing)

\[ = \min \left( \frac{x(21.5^2)}{4} \times 102.5, 21.5 \times 8 \times 236 \right) \]

\[ = \min \left( 37.2 \text{ kN}, \frac{50}{37.2} \right) \]

Rivet value = 37.2 kN

No. of rivet = \( \frac{50}{37.2} = 2 \) rivet adopt

for I = \( \frac{80}{37.2} = 3 \) rivet adopt
Location of connection for angle

Double shear

Fixed value for angle \( \text{III} \) = \( \min \left( \text{shearing, bearing} \right) \)

\[ = \min \left( \frac{2 \pi d^2}{4} \times 102 \times 0.01 \times 12 \times 0.0236 \right) \]

\[ = \min 74.42 \text{, } 21.5 \times 12 \times 0.0236 \]

\[ = 60.89 \text{ kN} \]

Net force needs to be transferred to the junct

\[ = 412 - 300 = 112 \text{ kN} \]

No. of rivet = \( \frac{112}{60.89} \) = Adopt 2 rivet

For thin shells

For thin shells the connection will be done per pitch

Length, force in one pitch length will be stress \( \times \) pitch length \( \times \) thickness

Stress will be for cylindrical shell

1) Hoop stress = \( \frac{F d}{2 \pi t} \)

2) Long stress = \( \frac{F d}{4 \pi t} \)

The force for which connection will be designed

\[ f = \text{shear} \times 5 \times \text{thickness} \]

\[ \frac{1}{\text{pitch length}} \]
To start with efficiency will be assumed and finally it will be shown that actual efficiency is more than the assumed value.

Note: If the rivet is subjected to combined shear and tension

\[
\frac{(\text{Shear stress})_{\text{calc.}}}{(\text{Permissible shear stress})_{\text{in rivet}}} + \frac{\text{Tensile stress calc.}}{\text{Per.}} \leq 1.4
\]

Eccentric Connection

- Column
- Bracket plate

\[ F_{di} = \text{force on the } i\text{th rivet due to direct loading} \]

\[ F_{di} = \frac{P \cdot A_i}{n \sum A_i} \]
Under the assumption that direct shear stress is equally shared.

Direction of this force will be parallel to the line of action of force.

We know that torsional shear stress = $\tau_i = \frac{T \delta_i}{J}$

Torsional shear force = $\frac{T \delta_i}{J}$

$F_{ti} = \frac{(pe) A_i \delta_i}{J}$

For discrete area $J = \sum A_i \delta_i^2$.

$F_{ti} = \frac{(pe) A_i \delta_i}{\sum A_i \delta_i^2}$

$(pe) \rightarrow$ Torsional Moment

$A_i \rightarrow$ Area of $i^{th}$ rivet

$\delta_i \rightarrow$ Distance of $i^{th}$ rivet from the C.G. of rivet group

The direction of this S.F. is perpendicular to the line joining the C.G. and the rivet under consideration and it will be in the same sense as that of the torsional moment.

Resultant force on the $i^{th}$ rivet

$F_{ri} = \sqrt{F_{di}^2 + F_{ti}^2 + 2 F_{di} F_{ti} \cos \theta}$

For safety of connection resultant force in all rivets should be less than their rivet value.

Note: To design the connection no. of rivets are chosen as follows:

...
If all the rivets are of same dia then most critical rivet is the one which is farthest from the C.G. If there are more than one rivets equally distant from C.G. then the rivet in which angle b/w $F_D + F_T$ is the most critical rivet.

If dia of all the rivets are same

$$F_{D_i} = \frac{p}{n}$$

$$F_{T_i} = \frac{(Pe) r_i}{\sum d_i}$$

1. 2, 3 & 4 have same value of $F_D$ and $F_T$ but angle b/w $F_D + F_T$ is smaller in $0 < \theta$ hence they are the most critical rivet.

1 & 2 are most critical rivet.
A bracket connected to the flange of a column through a group of rivet to support a load of 90 kN as shown in figure below. Thickness of bracket plate is 10 mm and that of flange of column is 8 mm. Determine the max force developed in the rivet and design a suitable rivetted joint.

Allowable stress in single shear = 100 MPa
in double shear = 200 MPa

\[ \sigma_{fr} = 180 \text{MPa} \]

Assume size of rivets to be

Some hence rivet 14.2 will become most critical rivet

Sol.

\[ F_{D_i} = \frac{90}{g} = 10 \text{ kN} \]

Hence let us calculate force in rivet 1 which is max

\[ F_{D_1} = \frac{90}{g} = 10 \text{ kN} \]

\[ F_{T_1} = \frac{P \Sigma \gamma_i}{\Sigma \delta^2_i} = \frac{90 \times 150 \times 100}{(4 \times 100^2 + 2 \times 80^2 + 2 \times 60^2)} \]

\[ = 22.5 \text{ kN} \]

\[ \cos \theta = \frac{3}{5} \]

\[ F_{r_1} = \sqrt{F_{T_1}^2 + F_{D_1}^2 + 2F_{T_1}F_{D_1} \cos \theta} = 29.6 \text{ kN} \]

\[ \frac{A}{4} (d + 1.5)^2 \times 100 \geq 29.6 \times 10^3 \]

\[ d \geq 14.4 \text{mm} \]

\[ 17.9 \text{mm} \]
Bearing strength of rivet: $d + 1.5 \times 8 \times 180 > 29.5 \times 10^3$

$d > 19.05 \text{ mm}$

hence adopt rivet dia. $= 20 \text{ mm}$

Rivets 1-6 are of dia $d'$
7th rivet is of dia $1.2d'$

Find force in 7th rivet

$\cos \theta = \frac{2}{\sqrt{5}}$

$sin \theta = \frac{1}{\sqrt{5}}$

$P = 10t = 100 \text{ KN}$

$\tan \alpha = \frac{1}{3.367}$

$\alpha = 26.565^\circ$

$\phi = (\alpha + \beta) = 60.255^\circ$

$x = A_1 x_1 + A_2 x_2 + A_3 x_3 + \ldots + A_n x_n$

$A_1 + A_2 + \ldots + A_n$

$= 0 + 0 + 0 + 3.65 A \times 80 + 1.44 A \times 160$

$7.44 A$

$= 63.226 \text{ mm}$

$y = 0 + 0 + 2 A \times 80 + 2 A \times 160$

$7.44 A$

$= 64.516 \text{ mm}$

$F_{D7} = \frac{100 \times 1.44 A}{7.44 A} = 19.355 \text{ KN}$

$(Pc) = \frac{100 \times 100 \times (80 + 80 - 63.226) + 100 \times (64.516)}{\sqrt{5}}$

$q = 115.81 \text{ KN/m}$

$R = 175.40 \text{ KN/mm}$

$c = 57.7 \text{ mm}$

$Pc = 5770 \text{ KN/mm} = 5.77 \text{ KN}$
\[ y_1 = 114.51 \text{ mm} \]
\[ y_2 = 96.94 \text{ mm} \]
\[ y_3 = 68.08 \text{ mm} \]
\[ y_4 = 22.88 \text{ mm} \]
\[ y_5 = 90.33 \text{ mm} \]
\[ y_6 = 26.67 \text{ mm} \]
\[ y_7 = 110.31 \text{ mm} \]

\[
\frac{(Pe) A_1 y_7}{\sum A_1 y_1^2} = \frac{(Pe) A(y_1 + \ldots + y_6) + 1.44 A x y_7}{A(y_1^2 + \ldots + y_6^2) + 1.44 A x y_7^2}
\]

\[ f_{a_1} = 16.28 \text{ kN} \]

\[ f_{a_2} = \sqrt{f_{d_1}^2 + f_{T_2}^2 + 2 f_{d_1} f_{T_2} \cos(60.25^\circ)} \]
\[ = 30.86 \text{ kN} \]
Welded Connection

Types of weld:

1) Butt weld or groove weld
2) Fillet weld
3) Plug weld
4) Slot weld

Welding is done by electric arc welding.
The various types of butt welds are square butt weld adopted up to \( t \leq 8 \text{mm} \)

Other types of weld are

- Single U - butt weld
  Normally up to 40 mm

- Double U - butt weld
  \( > 40 \text{mm} \)

Other types of butt weld are single V, single J, single double V, etc.

Butt weld is normally done in the workshop.

Partial penetration and complete penetration

When weld metal does not penetrate to the complete depth of the plate.

Weld metal penetrates completely to the full depth is known as complete penetration.

\[
P_{\text{max}} = B \times t \times A_{\text{t}}
\]

Partial or incomplete penetration

\[
P_{\text{max}} = B \times t' \times \delta_{\text{w}}
\]

If the thickness of penetration is not given, \( t' \) can be taken \( \frac{5}{8} \) times of \( t \text{mm} \), but it is not IS code recommendation.
Fillet Weld can be done in the field as lesser precision is required in it. It is cheaper as compared to butt weld. Fillet weld is almost always assumed to fail in shear.

Permissible Stresses

Tension and compression on section through the throat of butt weld

\[ \sigma_{km} = 0.6 \sigma_y \]

Shear of section through the throat of butt or fillet weld

\[ = 0.4 \sigma_y \] (Normally it is 100 to 110 N/mm²)

Then values are for welding done in workshop they are reduced to 80%. In case of field welding,

In case of wind or EQ the permissible values are increased by 25%.

Design of butt weld

- Butt weld or groove weld is more suitable for alternating stress provided that full penetration of weld is achieved.
- Reinforcement is good for static load condition. It is not suitable for alternating stress condition due to huge stress conc. at this location leading to onset of crack.
- However even in this case reinforcement of 0.6 to 0.75 is kept.
- The strength of weld in tension and compression is given by

\[ \sigma = \sigma_{eff} \times 0.6 \sigma_y \]

\[ L_{eff} - \text{Actual length of weld} \]

\[ L_{eff} = \text{Length of full size of weld} \]

\[ t_{eff} - \text{Thickness of thinner part being joined (mean of complete penetration)} \]
For incomplete penetration leff is min thickness of weld metal common to the parts being joined excluding reinforcement.

When two plates are of different thickness

\[ t_1 \quad t_2 \]

If \( t_1 - t_2 \) \( > \frac{t_2}{4} \) or 3mm whichever is greater, then we provide tapering in the thickness. The tapering should not be greater than 1 in 5.

If the thickness of weld common by which the plates is not given in incomplete penetration weld leff can be taken as \( \frac{5}{8} \) times \( t_{\text{min}} \).

**Design of Fillet Weld**

**Size of the Weld**

Size of the weld is decided on the basis of largest right angle that can be inscribed in the weld for normal fillet weld size is taken as the minimum weld leg size.

\[ \text{Size of weld} = s_1, \quad \text{Size of weld} = s \]
For right angle fillet weld minimum throat size will be \( \frac{S_1 S_2}{\sqrt{S_1^2 + S_2^2}} \). However, as per IS code it is taken as \( \frac{S_1}{\sqrt{2}} \).

Minimum size of weld:

Thickness of thicker part

- 0 - 10: 3 mm
- 10 - 20: 5 mm
- 20 - 32: 6 mm
- 32 - 50: 8 mm first pass, 10 mm final

Min size however should not be more than thickness of thinner plate being joined.

If the weld size is small it will cool faster due to heat extracted by the thicker plate. Hence rapid cooling leads to brittleness in the weld hence there should be min size of the weld controlled by the thickness of thicker plate.

Max size of weld:

For square edge, max size = thickness of thinner plate being joined - 1.5 mm.

Square edge.
for round edge

\[ \text{Max size of weld} = \frac{3}{4} \text{nominal thickness of round edge} \]

\[ 42 \]

**Note:**
from economy point of view it is better to use smaller size of longer length of weld, and longer.

\[ \text{Quantity of steel} = \frac{1}{2} r s x s x s L \]
\[ \propto L s^2 \]

\[ \text{4 Force} \propto L s \]

**Throat of fillet weld:**

Effective throat thickness is given by 
\[ = k \times \text{size of weld} \]

\[ \begin{align*}
\theta & \quad 60 - 90 \quad 91 - 100 \quad 101 - 106 \quad 107 - 113 \quad 114 - 120 \\
k & \quad 0.8 \quad 0.7 \quad 0.65 \quad 0.6 \quad 0.55 \quad 0.5
\end{align*} \]

Area resisting shear in fillet weld is \( 8 x t \)
where \( l \) is the length of weld, \( t \) is the effective throat thickness hence force resisted by fillet weld

\[ 8 x t \times 0.4 f y \quad \text{Permissible shear stress} \]

\[ l \] is throat thickness.
Effective length of fillet weld.

\[ L_{eff} = \frac{L}{2s} \]

where \( s \) is the size of weld.

In drawings, the length shown is the effective length.

Minimum length of weld = \( 9s \) (for weld to be 100% effective in load transfer).

Side fillet weld.

\[ p \quad \begin{array}{c}
\nearrow \\
\uparrow \\
\searrow \\
\downarrow \\
p
\end{array} \quad b \\
1 \\
\]

\( l \neq b \)

As width increases, non-uniformity in stress in the plate increases to make it more uniform, plug or slot weld can be used.

\[ \] 

Stress variation on the plate.

Intermittent fillet weld.

When the length of smallest size weld required to transmit load is less than the continuous length of joint, intermittent fillet weld is provided.

\[ \text{gap} \quad \text{(mean of arc or gap)} \quad \text{let} \quad 2 \text{mm} \quad \text{whichever is smaller} \quad t \quad \text{thickness of thinner plate} \]

[Diagram of intermittent fillet weld]
\[ P = 2xt \times 4fy \]  
Find \( t \) if \( l > (2a + t) \)  

The strength of end fillet weld is generally larger than that of side fillet weld however for strength calculation we take the strength of the two welds to be same.

Plug and slot weld.

Effective connection b/w top & bottom plate

Plug weld is capable of resisting when however as per codal recommendation it is not considered to be contributing to the strength.

Plug weld is also provided to make stress variation in top plate more uniform.
If the overlap is limited and even by providing the largest size of weld all along the available length, if force can not be resisted then we provide additional length of connection by making slot. The additional length of connection in the above figure will be 4t.

Width of slot $\leq 3t$ or $25$ mm whichever is greater.

Note: Min overlap in lap joint $\leq 4t$ or $40$ mm whichever is more.

Fillet weld for truss member.

Truss joint should be such that it should be moment free.

$$P_1 h_1 - P_2 h_2 = 0$$

$$\frac{P_1}{P_2} = \frac{h_2}{h_1}$$

$$\frac{L_1}{L_2} = \frac{h_2}{h_1}$$

$$\frac{L_1}{L_2} = \frac{h_2}{h_1}$$
\((h_1, h_2) \cdot t \cdot 0.4 \cdot f_y = P \quad \text{(11)}\)

From Eq. \((1) + (11)\), \(l_1 + l_2\) can be found out.

Case II

\[ P = (l_1 + l_2 + l_3) \cdot t \cdot 0.4 \cdot f_y \quad \text{(1)} \]

\(l_3\) is throat thickness.

\[ f_1, h_2 - f_2 h_2 - h_3 \left\{ \frac{(h_1 + h_2)}{2} - h_1 \right\} = 0 \]

\[ f_1 = f_2 = f_3 = t \cdot 0.4 \cdot f_y \]

Indian standard light channel.

An ISLC 300 with area 4211 \(\text{mm}^2\), \(T = 11.6\) \(\text{mm}\) (flange thickness), \(t = 6.7\) \(\text{mm}\) (web thickness) is used to transmit a pull of 600 \(\text{kN}\). Thus, the channel section is connected to a gird plate 10 \(\text{mm}\) thick. Design a fillet weld if overlap is limited to 350 \(\text{mm}\) and welding can be provided only on 3 sides. Use slot weld if required.
Max size of fillet weld on side 1 = Thickness of thinner part
- 1.5 mm (2p edge)

Min size

Max size of fillet on side 2 = 6.7 - 1.5 = 5.2 mm

If uniform size of weld is to be provided we will adopt 5 mm size.

Max force that can be resisted by providing weld on 3 side and adopting the max possible size.

\[ P_{\text{max}} = 350 \times 2 \times t_1 \times 0.4 f_y + 300 \times t_2 \times 0.4 f_y \]

Possible

\[ t_1 = 0.7 s_1 = 0.7 \times 8.5 \]
\[ t_2 = 0.7 s_2 \]

\[ P_{\text{max}} = 525.7 \text{ KN} \leq 600 \text{ KN} \]

Hence we need to provide slot welding.

Hence we will provide slots by adopting a uniform size of weld = 5 mm
Length of weld required = 1

\[ 600 \times 10^3 = 1000 \times 0.7 \times 5 \times 0.9 \times 250 \]

\[ l_{w} = 1714.28 \text{ mm} \]

Length of slot weld required = 1714.28 - 1000 = 714.28 mm

Drawing two slots

Length of slots required = \[ \frac{714.28}{2} = 357.14 \text{ mm} \]

Adopt 180 mm

Width of slot = 30 or 25 mm whichever is more

= 3 \times 6.7 or 2.5

= 20.1 or 2.5

Take b = 25 mm

6. In an industrial shed and edge support consisting of two angles 110 x 110 mm is to be connected to 16 mm gusset plate for a tensile load of 650 KN.

Design the moment free welded connection.

\[ M = \frac{650 \times 120}{2} = 39000 \text{ KN} \cdot \text{mm} \]

\[ \theta = \frac{60}{180} = 3.43 \degree \]

Area of slot height = 2 \times 1.5 = 3 mm

\[ N_{EA} \text{ (stress twisted by moment) = (650 - 2.5 \times 3.43 \times 1.5) \geq 3.96 \times 10^3} \]

\[ 290 = \frac{l^2}{2} \times 156 = 44 \times 10^3 \]

\[ l > 10.33 \text{ mm} \]

Take l = 12 mm
The best arrangement for transfer of loading to the gusset plate will be when two angles are connected on off side of gusset plate.

Hence each angle will carry 325 KN load.
Hence we have to decide length of weld, size of weld and arrangement of weld.

As the thickness of angle is not given, we will proceed on the basis of min size of weld decided on the basis of thickness of gusset plate.

Min size = 5 mm adopted

Length of weld req. = \( \frac{325000}{5 \times 0.7 \times 100} \) = 928.57 mm

Notes:
1. Max thickness available for angle
2. Max size available for angle
   a) Equal angle
   b) Unequal angle
3. Max depth of I section available
(iv) Gauge location for I-section
(v) Min thickness of plate
(vi) Max thickness of plate

\[ \text{Strength of weld} = 0.7 \times 5 \times 100 = 350 \text{ N/mm} \]

\[ f_1 + f_2 = 928.57 - 110 = 818.57 \text{ mm} \]

\[ P_1 = \frac{l_1 \times 10 \times 4 \text{ fy}}{2} \]

\[ P_2 = \frac{l_2 \times 0.75 \times 0.4 \text{ fy}}{2} \]

\[ P_3 = \frac{l_3 \times 0.75 \times 0.4 \text{ fy}}{2} \]

\[ P_1 = 325 - 38.5 = 286.5 \]

For moment free connection

\[ P_1 h_1 = P_2 h_2 + P_3 \left( \frac{h_1 + h_2}{2} - h_1 \right) \]

\[ 27.1 \]

\[ 79.1 \]

\[ 30.9 \]

\[ v \]

\[ w \]

Weld Notations

\[ \triangle \text{ Fillet weld} \]

\[ \square \text{ Square butt weld} \]

\[ \checkmark \text{ Single V} \]

\[ \times \text{ Double V} \]
All around welding

Field weld

Field butt weld of throat thickness = 6mm

and weld is double bevel.

5 mm shop fillet weld of length 125 mm

5 mm weld, l = 150 mm

8 mm weld, length 200 mm

5 mm weld, shop weld.
Q. A plate of 150 mm width and 20 mm thickness is welded to another plate by fillet weld as shown in figure the size of weld is 12 mm throughout compute the av. shear stress produced in the weld for the full strength of plate if the allowable stress is 150 N/mm² in axial tension.

Note: As in this case the location of slot is not given the strength of plate can not be decided hence we will work on full strength of plate at see 1-1.
Full strength of plate = 150 \times 10 \times 150 = 450 \text{KN}

Total length of weld = 2 \times (150 + 70.72) + 50 + 7 \times 60 = 680 \text{mm}

Note: Strength of plug weld is not considered in design but we can consider the strength of root weld.

\[ f_s = 450 \times 10^3 \]

\[ f_s = \frac{450 \times 10^3}{680 \times 0.7 \times 12} = 78.78 \text{ N/mm}^2 \]

E9

A circular shaft of diameter 150 mm is welded to a rigid plate by an external all round fillet weld of size 10 mm if a torque of 10 KNm is applied to the shaft find the maximum stress in the weld.
Shear stress = \( \frac{TY}{J} \) - Polar moment of inertia of resisting section

Exact Calc. of \( J \)

\[
J = \int_0^{7.5} 2\pi (30 + \frac{x}{\sqrt{L}})^3 \, dx = 20.474 \times 10^6 \text{ mm}^4
\]

\[
I_{max} = 7.5 + 0.7 \times 10 = \frac{79.949}{\sqrt{2}} \text{ mm}^4
\]

Max. shear stress = \( \frac{10 \times 10^6 \times 79.949}{20.474 \times 10^6} = 39.05 \text{ N/mm}^2 \)
Approximate calc.

\[ J = \frac{\pi}{32} \left[ \left( 150 + 2 \times 4.9 \right)^4 - 150^4 \right] \]

\[ = 14.03177 \times 10^6 \text{ mm}^4 \]

\[ T = \frac{TY}{J} = \frac{10 \times 10^6 \times 79.95}{14.03177 \times 10^6} = 55.83 \text{ kN/mm}^2 \]

\[ J = \frac{\pi}{32} \left[ \left( 150 + 14 \right)^4 - 150^4 \right] = 21.318 \times 10^6 \text{ mm}^4 \]

Approximate calc. is quite different from the actual value a
logical calculation will be when the projected width is taken
as the actual width of resisting section and it is assumed
that the stress is uniform over the cross section of thrust

To shear stress on thrust
assuming constant

\[ T_0 = \frac{10 \times 10^6}{2 \times 10 + \frac{7.5}{5}} = 35.57 \text{ kN/mm}^2 \]

\[ = 0.475 \]
Eccentric connection using weld.

Rivet 2 is subjected to shear and axial tension.

Subjected to direct shear and torsional moment.

Tension in a rivet will be corresponding to the value in bending stress diagram.

In rivet 2 check for safety is done as follows:

\[
\frac{\text{Calculated axial force}}{\text{Permissible axial force}} + \frac{\text{Calculated shear force}}{\text{Permissible shear force}} \leq 1.5
\]
Eccentric connection using welding.

Case 1

Weld subjected to direct shear + Torsional shear

\[ f_t = \sqrt{f_d^2 + f_e^2 + 1.66f_e f_d \cos \theta} \leq f_{	ext{per}}. \]

Shear stress in weld

Size of weld = 5

As welding is continuous area we will work in terms of stress

\[ f_d = \frac{P}{(2a+d)t} \]

Throat thickness

\[ f_d = \frac{P}{(2a+d)(a-7.5)} \]

direct shear on throat of weld.

Direction of \( f_d \) is taken along the line of action of force \( P \)

\[ f_t = \frac{Ts}{J} \]

Weld will be considered as a line area as shown above. Thickness of the line will be assumed as throat thickness

\[ I_\rho = I_{xx} + I_{yy} \]
\[ I_{xx} = \frac{td^3}{12} + (\alpha - t)(\frac{d}{2})^2 x^2 \]

\[ I_{yy} = (txd)x^2 + \left[ \frac{ka^3}{12} + a t \left( \frac{a}{2} - \frac{x}{2} \right)^2 \right]^2 \] 

**Case I**

Distance of point under consideration from c.g. of weld.

**e** is perpendicular distance of line of action of p from the c.g. of weld.

Direction of ft will be same to the line joining point under consideration to the c.g. of weld and it will be in the same sense as that of the torsional moment.

Resultant shear stress:

\[ f_v = \sqrt{f_{d1}^2 + f_{t2}^2 + 2f_{d1}f_{t2} \cos \theta} \]

For safety

**Case II**

Fillet weld subjected to direct load and bending.

- Column
- Beam
Direct shear stress
vertical
\[ f_v = \frac{P}{2dt} \]

At any point along the length of weld the bending stress will be resisted by the fillet weld through shearing action on its throat hence horizontal shear stress in fillet weld \( f_h \)

\[ f_h = \left( \frac{M_y}{I_{xx}} \right) \]

\[ I_{xx} = 2 \left( \frac{td^3}{12} \right) = \frac{2(0.75)d^3}{12} \]

\[ f_s = \sqrt{f_h^2 + f_v^2} \]

For safety of connection \( f_s < \) permissible shear stress in weld.

Case III
Butt Weld

Butt weld is just an extension of plate whatever is the stress in the plate, it will be resisted by butt weld

\[ f_s = \text{shear stress in butt weld} = \frac{P}{td} \text{ (vertical)} \]
\[ f_b = \text{bending stress in butt weld} = \frac{\text{PE of member}}{I_{yy}} = \frac{\text{PE (d/2)}}{t d^2/12} \]

Checking safety using interaction formula

\[
\left( \frac{f_s}{\text{sum. stress in butt weld}} \right)^2 + \left( \frac{f_b}{\text{perm. bending stress in butt weld}} \right)^2 \leq 1.0
\]

Equivalency method -

\[
\sqrt{f_b^2 + 3 f_s^2} \leq 0.9 f_y
\]

Based on max. distortion energy theorem

Permissible bending stress for flanged section = 165 \( \text{N/mm}^2 \)

(0.67 \( f_y \))

For solid sections: \( \boxed{\square, \bigcirc, \bigtriangleup} \) permissible

Bending stress is 185 \( \text{N/mm}^2 \)
A welded bracket connects the a plate to a column flange as shown in the fig. below determine the size of weld if the allowable stress in the weld is 110 N/mm².

\[ x' = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2 + A_3} \]

\[ = 2 \left(150 \times t \times 75\right) + 0 \]

\[ \frac{1500 \times 10^3 + 0}{550 \times t} = 40.91 \text{ mm} \]

Maximum stress will be either at A or at D.

At A:

\[ \sigma_d = \frac{100 \times 10^3}{550 \times t} = \frac{181.818}{t} \text{ N/mm}^2 \]

\[ \sigma_t = \frac{\tau_T}{J} = \frac{\tau_T}{I + \tau_T} \]

\[ I_x = 2 \left(150 \times t \times 125^2\right) + t \frac{250^3}{12} = 5.9876 \times 10^6 \text{ t mm}^4 \]

\[ I_{yy} = 250 t \times x^2 + \left[\frac{150 t}{12} + \left(t \times 150 \times (75 - 40.91)^2\right)\right] \times 2 = 1.3295 \times 10^6 \text{ t mm}^4 \]
\[ T = 100 \times 10^3 \left( 300 - 40.91 \right) = 25.9 \times 10^6 \text{ N}\text{mm} \]

\[ I_N = \frac{(150 - 40.91)^2}{125^2} = 165.908 \text{ mm}^4 \]

\[ f_t = \frac{T_{\text{at}}}{J} = \frac{587.8076}{t} \text{ N/mm}^2 \]

\[ \cos \theta = 0.657 \]

\[ f_r^2 = f_d^2 + f_t^2 + 2f_df_t \cos \theta \]

\[ f_r = \frac{720.009}{t} \text{ N/mm}^2 \]

For safety of connection, \( f_r \leq \text{per. shear stress} \)

\[ \frac{720.009}{t} < 110 \]

\[ t > 6.545 \text{ mm} \]

\[ s > 6.545 \frac{t}{f} \]

\[ > 9.35 \text{ mm} \]
Weld is subjected to direct shear \( W \)
and \( BM = W \)

Thus external things are resisted by fillet weld through shearing action

Direct shear stress
\[
\sigma_{\text{direct}} = \frac{W}{1000 \times 7.96 + 120 \times 7.93} \times \frac{N}{\text{mm}^2}
\]
\[
= \frac{W}{1344} \text{ N/mm}^2
\]

Shear stress
\[
\tau = \frac{M_y}{I} = \frac{W(200) \times 1000}{I}
\]

\[
I = 2 \times 100 \times 7.96 \times 100^2 + \frac{7 \times 3 \times 120^3}{12}
\]
\[
= 3.0048 \times 10^6 \text{ mm}^4
\]

\[
f_x = \frac{W}{450.24 \times 1000000} \text{ N/mm}^2
\]

\[
f_r = \sqrt{f_x^2 + f_t^2} = \sqrt{\left(\frac{W}{1344}\right)^2 + \left(\frac{W}{450.24}\right)^2}
\]
\[
= 2.342 \times 10^{-5} \leq 0.01
\]
\[
W \leq 92.69 \text{ kN}
\]

Note: Welding system exist lateral loads.
Comparison between tension and compression member.

**Tension Member**

1) Net area is effective.
2) There is no stability problem.
3) Permissible stress $\sigma_{at} = 0.6 \, f_y$
4) Design is straightforward.

**Compression Member**

1) Gross area is effective.
2) There is stability problem, hence member may buckle before achieving full strength.
3) $\sigma_c < 0.6 \, f_y$
4) Design is based on tabulated values.

Note: Theoretically there is no limitation on slenderness ratio of tension member since stability is of little concern. However, the member may be subjected to compressive load during transportation and erection, hence in order to provide adequate rigidity to prevent undesirable lateral buckling and excessive vibration, IS code limits the slenderness ratio values as follows:

$$\text{Slenderness ratio } \lambda = \frac{L}{r_{min}}$$

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Description</th>
<th>Max. S.t.</th>
<th>Max S.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>A tension member in which reversal occurs due to load other than wind or EQ</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>A member normally acting as a tie in a roof truss or bracing system but subjected to compression due to wind or EQ</td>
<td>350</td>
<td></td>
</tr>
</tbody>
</table>
Net Sectional Area

For Plates:

A plate may have various modes of failure in tension hence all the modes are required to be examined and the mode which gives the min net area will be the most critical mode.

\[ A_{\text{net}} = Bt - nd't + \left( \frac{b_1^2}{4g_1} + \frac{b_2^2}{4g_2} \right) t = B_{\text{eff}} t \]

No. of rivets along failure line for each inclined line \( \left( \frac{b_2}{4g_2} t \right) \) is added.

\[ A_{\text{net}} f_{n'y} = Bt - 2d't + \frac{b_2^2 t}{4g_1} \]
The thickness of the plate is 10 mm. Find the area provided by the hole.

\[ \text{Area}_{\text{ABC}} = 360 \times 10 - 2 \times 22 \times 10 = 3160 \text{ mm}^2 \]

\[ \text{Area}_{\text{ABFCD}} = 360 \times 10 - 3 \times 22 \times 10 + \left(\frac{50}{4 \times 60} + \frac{50}{4 \times 180}\right) \times 10 = 3078 \text{ mm}^2 \]

\[ \text{Area}_{\text{ABFGCD}} = 360 \times 10 - 4 \times 22 \times 10 + \left(\frac{50}{4 \times 60} + \frac{50}{4 \times 60}\right) \times 10 = 2928.33 \text{ mm}^2 \]

\[ \text{Area}_{\text{ARFGH}} = 360 \times 10 - 3 \times 22 \times 10 + \frac{50}{4 \times 60} \times 10 = 3044.17 \text{ mm}^2 \]

In this case for calculation of force we have to add the rivet value of c.

Hence the most critical section set is AB F C D.
Force is transferred in the member through the gusset plate. The location of transfer is at the location of rivets/welds. This force is then distributed in the whole member through shearing. The connected legs leads over the outstanding leg in the distribution through shear stress, thus there is a shear lag existing between connected and outstanding leg. The stresses in the angle at the location of joint can be shown as follows:

\[ A_1 = (l_1 - d' - \frac{t}{2}) + t \]

\[ A_2 = (l - \frac{t}{2}) + t \]

As the stress in outstanding leg is less, the force carried by it will be less to account for this net area is taken as \((1+K)A_2\)

where \(K\) is a reduction factor for outstanding leg

\[ A_1 \rightarrow \text{Area of connected leg} \]

\[ A_2 \rightarrow \text{Area of outstanding leg} \]
Area \( = \left( l_1 - \frac{t}{2} \right) t \)

Total area \( = \left( l_1 + l_2 - t \right) t \)

Single angle connected only through one leg only

\[
\begin{align*}
A_{net} &= A_1 + k \cdot A_2 \\
\kappa &= \frac{3A_1}{3A_1 + A_2} \\
A_1 &= \left( l_1 - d' - t/2 \right) t \\
A_2 &= \left( l_2 - t/2 \right) t
\end{align*}
\]

\[
\begin{align*}
A_{net} &= A_1 + k \cdot A_2 \\
\text{min of} \left\{ \left( l_1 - \frac{t}{2} - d' \right) t, \left( l_1 - \frac{t}{2} - 2d_1 - \frac{v^2}{4g} \right) t \right\}
\end{align*}
\]
Case-III (a) When two angles are connected on the same side of gusset plate (and are tacked)

\[ A_{net} = A_1 + KA_2 \]
\[ K = \frac{5A_1}{5A_1 + A_2} \]

- \( A_1 \) = area of connected legs
- \( A_2 \) = outstanding legs

(b) When two angles are connected on the same side of gusset plate and not tacked riveted.

In this case two angles will behave individually hence net area would be twice the area corresponding to single angle connected to a gusset plate.

Case-III (b)

Two angles connected on the gusset plate and tacked along the length.
Area = \left( A_{gusset} - \text{area of hole} \right) \\
= 2(l_1 + l_2) - \text{area of hole} \\
= 2(l_1 + l_2 - t) - 2d't \quad \text{(7)}

(8) When there is no tack riveting along the length.

The two angles will behave individually and net area would be twice that corresponding to single angle connected to gusset plate.

Note: Shear lag effect is non-existent in this case (cawd)

D. 09.63: An I Sar 75x75x10 is connected to gusset plate by 16 mm diameter rivets through both legs the pitch on each leg is 80 mm, and the rivets on one leg are staggered by 40 mm w.r.t. those other. Find allowable tension load on the angle.

[Diagram of the angle structure]
If both the legs are connected legs the angle can be considered as a plate.

\[ d^2 = 16 + 1.5 = 17.5 \]

\[ \text{Area of leg 1-1} = 140 - 12.5 \times 10 = 12.25 \text{ mm}^2 \]

\[ \text{Area of leg 2-2} = (140 - 2 \times 17.5 + \frac{40^2}{2 \times 80}) \times 10 = 1100 \text{ mm}^2 \]

\[ \text{Area of web} = 1100 \text{ mm}^2 \]

\[ \text{Poisson's ratio} = 1.100 \times 0.6 \text{ ft} = 65 \text{ KN} \]

Q. An ISA 150 x 115 riveted on one side of a gusset plate by two rows of 22 mm dia rivet through 150 mm leg. It is required to carry a tensile force of 320 KN. Find the thickness of the angle required. The diagram is as shown below.
\[ A_{\text{net req.}} = \frac{320}{0.6 \text{ fy}} = \frac{320 \times 10^4 \text{ N}}{150 \text{ N/mm}^2} = 2133.33 \text{ mm}^2 \]

\[ A_{\text{net available}} = A_{1\text{ net}} + \frac{1}{2} A_2 \]

\[ A_2 = \left(115 - \frac{t}{2}\right) \times t \]

\[ A_{\text{net}} = 150 - \frac{t}{2} - 2 \times 13.5 \times 40 \frac{1}{4 \times 60} \times t \]

\[ = \left(115 - \frac{t}{2}\right) t \left(109.67 - \frac{t}{2}\right) t \]

The thickness will be assumed in such a way that more than net area available should be more than net area required.

Assume \( t = 8 \text{ mm} \)

\[ A_{\text{net}} = 845.336 \text{ mm}^2 \]

\[ A_2 = 888 \]

\[ k = 0.74 \]

\[ A_{\text{net available}} = 1734.08 < A_{\text{net req.}} \text{ not safe} \]

Assume \( t = 10 \text{ mm} \rightarrow \text{ unsafe} \)

\[ t = 12 \text{ mm} \rightarrow \text{ safe} \]
\[ K = \frac{3A_1}{3A_1 + A_2} = \frac{3(109.07 - \frac{t}{2})}{3(109.07 - \frac{t}{2}) + (115 - \frac{t}{2})} \]

\[ NEA = A_1 + K A_2 \]

\[ 2135.33 = \left(109.07 - \frac{t}{2}\right) t + \frac{387.21 - 1.5t}{992.21 - 2t} (115 - \frac{t}{2}) t \]

\[ t = 11.59 \text{ mm} \]

take \( t = 12 \text{ mm} \)

Lug Angles

Lug angle is a short length of angle section used at a joint to connect the outstanding leg of main member thereby reducing the length of joint.
The rivet connecting the outstanding leg of main member with the lug angle should start in advance of all other rivets; this is done to ensure that force in the outstanding leg is effectively transferred to the lug angle when angle members are main members.

Lug angle and their connection with the gusset plate are designed for forces greater than equal to 1.2 times the force in outstanding leg of the main member.

Force in the outstanding leg: \[ F_{A2} = \frac{FA_2}{(A_1 + A_2)} \]

- \( A_2 \) = Area of outstanding leg
- \( A_1 \) = Area of connected leg

Hence designed force for lug angle and rivet 2 is

\[ F_{D2} = 1.2 \left( \frac{FA_2}{A_1 + A_2} \right) \]

Connection of outstanding leg of main member with the lug angle should be designed for force \( > 1.4 \times \) force in outstanding leg

Rivet 1 will be designed for

\[ F_{D0} = 1.4 \left( \frac{FA_1}{A_1 + A_2} \right) \]
When channel sections are main members:

1. Lug angles and its connection with the gusset plate should be designed for the force 1.1 times force in outstanding legs.
2. Connection of lug angles with the outstanding legs of channel should be designed for a force greater than equal to 1.2 times the force in the outstanding legs.
3. Minimum no. of rivets in lug angles should be 2.
4. Rivet 3 should be designed for force in connected leg.

\[ F_{\text{max}} = (A_{\text{gross}} - A_{\text{hole}}) \times \sigma_{\text{at}} \]

\[ A_{\text{gross}} = (75 + 100 - 10) \times 10 = 1650 \text{ mm}^2 \]

\[ A_{\text{hole}} = 0.785 \times 21.5 \times 10 = 215 \text{ mm}^2 \]

\[ F_{\text{max}} = (1650 - 215) \times 150 = 215,250 \text{ KN} \]

\[ A_1 = \left( \frac{100 - t}{2} \right) \times 10 = 950 \]

\[ A_2 = \left( \frac{75 - 5}{2} \right) \times 10 = 700 \]
Sent standing = \( \frac{FA_2}{A_1-A_2} \) = \( \frac{215.25 \times 790}{1650} \) = 71.21 KN

\( A_1 = (100 - 5) \times 10 = 950 \text{ m}^2 \)
\( A_2 = 700 \text{ mm}^2 \)

\( F_{\text{con}} = 123.93 \text{ KN} \)

Rivet value = \( \min \left( \text{Shearing, Bearing} \right) \), Bearing

\( k_r = \min \left( \frac{\pi d^2}{4}, d \times 300 \right) \)

\( = \left( \frac{\pi 21.5^2}{4}, 21.5 \times 300 \right) \)

\( = (36.305 \text{ KN}, 64.5 \text{ KN}) \), 36.3 KN

\( k_r = 36.305 \text{ KN} \)

No. of rivet 2 = \( \frac{123.932}{36.305} \) = 3.41

Adopt 4 no.

No. of rivet 2

Design of lug angle

Foreign lug angle = 91.518 \times 1.2 = 109.582 \text{ KN}

Actual req. = \( \frac{109.582 \times 10^3}{150} \) = 730.55 \text{ mm}^2

Let us adopt ISA 60\times60\times8

Actual = 896 - 21.5 \times 8 = 724 \text{ mm}^2 < \text{Req.}

Not safe

Hence adopt a bigger section
Let us adopt B.A. 60x60x10

And provided = 1100 - 21.5x10 = 885 mm² > And

No. of rivet 1 = \( \frac{1.4 \times Fea \times l}{R_u} \)

= \( \frac{1.4 \times 91318}{36305} \) = 3.52

Adopt 4 no

No of rivet 2 = \( \frac{1.2 \times Fea \times l}{R_u} \)

= \( \frac{1.2 \times 91318}{36305} \) = 3.018

Adopt = 4 no
The thickness of main plate depends on the tearing strength of the main plate that is

\[ \text{tearing strength} \geq 500 \text{ KN} \]

Tearing strength can be known only if arrangement of rivet is known.

Arrangement of rivet is known only when no. of rivets are known.

No. of rivets are known only when rivet value is known.

Rivet value is known only when we know whether shearing or bearing govern.

The rivets are in double shear hence shearing strength is known.

Bearing strength will depend on the thickness of main plate by equating shearing and bearing strength of rivet we get the thickness of main plate under the assumption that combined thickness of cover plate is more than that of main plate.

If thickness is not capable of resisting 500 KN load, thickness has to be increased. If thickness is increased bearing strength will become more than the shearing strength of rivet.

Hence the rivet value will be governed by shearing strength of rivet. Hence no. of rivet will be \( \frac{500}{\text{value}} \).
No. of rivet will be arranged and tearing strength a will be calculated at various sections in terms of thickness t

Taking per shear = 100 N/mm²

Shearing stress of rivet = \( \frac{2.5}{4} \times (2.5)^2 \times 100 \)

= 86.75 N

Assuming the thi combined thickness of cover plate to be more than main plate

bearing strength = \( d' \times t \times b' \times 9 \text{ mm}^2 \)

= 23.5 \times 300 \times t = 7.05 t \text{ kN}

86.747 = 7.05 t

\( t = 12.305 \text{ mm} \)

Max. force that can act area corresponding to this thickness can resist = 0.6 \times 12.305 \times 250 \times 260 = 480.8 \text{ kN} \leq 500 \text{ kN}

Hence thickness of main plate has to be ↑

Thus shearing strength govern the rivet value

\( \Delta \) \( b'v = 86.748 \text{ kN} \)

\( n = \frac{500}{86.748} = 5.7 \)

Adopt \( n = 6 \) rivet

The 6 no. of rivet can be arranged in diamond pattern
Tearing strength of steel 1:1

\[
(2.50 - 2.35) \times t \times 0.6 \times 260 \geq 500 \times 10^3
\]

\[
t \geq 14.746 \text{ mm } 14.741 \text{ mm } \text{(8)}
\]

\[
(2.50 - 2 \times 2.35) \times t \times 6 \times 260 + 86.748 \times 10^3 \geq 520 \times 10^3
\]

\[
t \geq 13.572 \text{ mm } 13.05 \text{ mm }
\]

Hence we will adopt thickness 15 mm

Let us adopt 16 mm thickness of main plate

At crest for strength of cover plate

For cover plate sec 3:3 is critical

\[
(2.50 - 3d') \times t \times 6 \times 260 \geq 500
\]

\[
t \geq 17.85 \text{ mm }
\]

Adopting thickness of cover plate as 10 mm each

For gusset plates as the thickness of main gusset plate as well as combined thickness of two cover plate are more than 12.3 mm hence shearing will govern the rivet value.

\[
\text{hence } \frac{500}{86.74} = 5.7
\]

Adopt 6 rivet
1) Tackling along a, b, c, d
\[ A_{net} = (A_{gray} - 4 \times \text{hole}) \]

2) Tackling along a + b
\[ A_{net} = \left[ 2 \times \begin{array}{c} \text{-} \text{tackling} \\ \text{-} \text{tackling} \end{array} \right] \]
\[ k = \frac{5A_1}{5A_1 + A_2} \]

3) Tackling along c + d
\[ A_{net} = \left[ 2 \times \begin{array}{c} \text{-} \text{tackling} \end{array} \right] \]
\[ k = \frac{6A_1}{5} \]

4) No tackling
\[ A_{net} = 4 \times \]
\[ k = \frac{3A_1}{3A_1 + A_2} \]
Ex.

Angle 60° x 60° x 10

Area = (60 + 60 - 10) x 10 = 330 mm²

Case 1

4 x 1100 - 4 x 21.5 x 10 = 3540 mm²

Case 2

k = \frac{5A_1}{5A_1 + A_2} = \frac{5 \times 670}{5 \times 670 + 1100}

\text{Area} = 2 \left( 670 + 7528 \times 1100 \right) = 2196.16 mm²

Case 3

2 \times \left( \frac{4}{5} \right) = 2 \times \left( 2 \times 1100 - 2 \times 21.5 \times 10 \right)

= 3540 mm²

Case 4

4 \times 10 = \left( A_1 + A_2 \right)

A_1 = 335

A_2 = \left( 60 - \frac{1}{2} \right) \times 10 = 550

k = \frac{3A_1}{3A_1 - A_2} = 0.6463

\text{Area} = 2761.86 mm²

Note: Max. deduction due to web hole at any section along the length of member should never be more than max. deduction of hole at any seen at the joint. This is done to ensure that if the joint is not failing member will not fail.
For long columns under purely axial loading the failure is always by buckling and buckling load is given by Eular's load

$$p = \frac{\pi^2 \varepsilon l}{L^2}$$

The buckling stress is given by Eular's stress

$$\sigma_{eu} = \frac{\pi^2 \varepsilon}{d^2}$$

$$d = \frac{I_{eff}}{I_{min}}$$

$\varepsilon_{min}$ = Min. radius of gyration of the section and $I_{eff}$ is Eff. length of compression member which depend on end support condition. However the IS code takes into account the failure by buckling and crushing simultaneously through the use of Merchant Hankine formula

$$\sigma_{eu} = \frac{6 f_y}{5 + \left(\frac{f_y}{f_{cu}}\right)^{1.9}}$$

$\sigma_{eu}$ = Eular stress

$\varepsilon_{min}$ = Min. radius of gyration of the section

$f_y$ = Yield stress

For a given grade of steel $\sigma_{eu} = f(d)$
As \( L \) decreases

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Fixed at one end and at other end restrained against rotation but fixed in position</td>
</tr>
<tr>
<td>(2)</td>
<td>Fixed at one end but at other end partially restrained against rotation</td>
</tr>
</tbody>
</table>

Note:

Length adopted by IS code is more than the theoretical value because it is impossible to simulate perfect fixity of support. So we take half of it.

1. To 1.5L

The above values are for a battened column.
Effective length of angle strut

<table>
<thead>
<tr>
<th>Description</th>
<th>$l_{eff}$</th>
<th>Allowable compressive stress $(\sigma_{ac})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous Single or double angle</td>
<td>$0.7l - l$</td>
<td>$\sigma_{ac}$</td>
</tr>
<tr>
<td>Discontinuous Single angle connected with one rivet only</td>
<td>$l$</td>
<td>$0.8\sigma_{ac}$</td>
</tr>
<tr>
<td>Single angle with more than one rivet or weld</td>
<td>$0.85l$</td>
<td>$\sigma_{ac}$</td>
</tr>
<tr>
<td>Double angle back to back, on opposite side of gusset plate</td>
<td>$0.7l - 0.85l$</td>
<td>$\sigma_{ac}$</td>
</tr>
<tr>
<td>Double angle on same side of gusset plate</td>
<td>$l$</td>
<td>$0.8\sigma_{ac}$</td>
</tr>
</tbody>
</table>

*Actual value of effective length depend on the rigidity of joint.*

[Diagram showing continuous and discontinuous joints]
Maximum Slenderness Ratio

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Description</th>
<th>$A_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Member carrying compressive load due to dead &amp; live load only</td>
<td>180</td>
</tr>
<tr>
<td>(2)</td>
<td>Member subjected to compression due to wind or EQ</td>
<td>250</td>
</tr>
<tr>
<td>(3)</td>
<td>For compression flange of beam</td>
<td>300</td>
</tr>
<tr>
<td>(4)</td>
<td>Member normally acting as a tie in a roof truss or bracing system but subjected to reversal of stress due to wind or EQ</td>
<td>350</td>
</tr>
</tbody>
</table>

![Rolled Section Diagram]

\[
\delta_{min} = \varepsilon_{uv}
\]

\[
\delta_{min} = \gamma_{uv}
\]
for rolled sections M.O.I. can not be changed. Hence to find out the load carrying capacity $I_{\text{min}}$ needs to be located. If the section is having at least one axis of symmetry, $I_{\text{min}} = \sqrt{\frac{I_{\text{min}}}{A}}$ will be about the axis of symmetry or about an axis $\perp$ to the axis of symmetry. If however the section is having no axis of symmetry then major and minor principal axis needs to be located.

In case of built up sections we can change the M.O.I of combined section $\alpha$

![Diagram of cross-section with labeled axes and moments of inertia]

$\gamma$ Axis of symmetry

\[I_{xx} = 2 \times I_{xx} \text{ of one section}\]

\[I_{yy} \text{comb.} = 2 \left[ I_{yy} \text{ori.} + A \left( \frac{Cy + S}{L} \right)^2 \right] \]

If spacing b/w seen is zero $I_{yy} \text{comb.}$ in this case will be less than $I_{yy}$ of combination hence $I_{\text{min}}$ will be $I_{yy}$ of combination.

If $I_{\text{min}}$ is less $I_{\text{min}}$ will be less so for this $A$ will be more and hence $\sigma_w$ will be less.
Thus load carrying capacity will be less.
To increase the load carrying capacity for the given area, $I_{xx}$ of combination needs to be increased. This can be achieved by increasing the spacing. For a given spacing $s_0$, $I_{xx}$ of combination will become $I_{yy}$ of comb. for $s > s_0$, $I_{min} = I_{xx}$ of comb. which is fixed. Hence load carrying capacity becomes fixed for all $s > s_0$ and this spacing corresponds to maximum load carrying capacity for a given sectional area. Thus for best utilization of material $b_{xx}$ of comb $= b_{yy}$ of comb.

Radius of gyration of a sections will not change when put in combination when the axis does not shift.

---

[Diagram 1]

---

[Diagram 2]
for a given overall size of column strength of \( f \) will be more than the strength of \( II \) because shifting of axes will be more in (I) than in (II) - 
\[ d > d' \]
for maintenance point of view \( II \) will be better.

\# ES Code Recommendation

\[ 90 \]

\[ \frac{A_{welded}}{A_{(\text{min})\text{comb}}} \]

\[ \text{Spacing} = \frac{\text{Spacing by}}{\text{Spacing rivet}} \]

\[ \text{Spacing} = \frac{\text{Spacing by}}{\text{Spacing rivet}} \]

When two components are placed back to back they should be tack rivetted such that stress concentration of individual section by/with tack rivet should not be
more than 40 nor more than \( -0.6 \times \) slenderness ratio
of whole seen

\[ \text{Dove} \neq 40 \]

\[ \neq 0.6 \times \text{whole seen} \]

This recommendation safe guards against local buckling of
individual sections between tacking rivets.

The spacing b/w tacking rivet however should not be
more than 600 mm.

The dia of tacking rivet should not be less
than the min dia given below.

<table>
<thead>
<tr>
<th>Thickness of member</th>
<th>Dia of tacking rivet up to</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 10 \text{ mm} )</td>
<td>18 mm</td>
</tr>
<tr>
<td>( 10 \text{ mm} &lt; 16 \text{ mm} )</td>
<td>20 mm</td>
</tr>
<tr>
<td>( &gt; 16 \text{ mm} )</td>
<td>22 mm</td>
</tr>
</tbody>
</table>

**Note**: Tacking rivet recommendation given before
32 t or 500 mm is the max. Spacing b/w tacking
rivets in any position. However the limiting
criteria in compression will become

\[ \text{Dove} \neq 40 \]

\[ \neq 0.6 \times \text{whole seen} \]

- Purpose of tacking rivet is to hold the two member together
- and to equalize the stress in the component member
If the web of channel is \( >150 \text{ mm} \) in depth, two rows of tackless rivets can be provided. If angles are 125 mm or more, two rows of tackless rivets can be provided.

8.10 1.49

\[
I_{yy} = 2 \times 150 \times 828 \times 10^3 + \left[ \frac{500 \times 10^3}{12} + \frac{500 \times 10 \times 2.05^2}{12} \right] \times 2
\]

\[
= 721.9893 \times 10^6 \text{ mm}^4
\]

\[
I_{yy} = 2 \left[ 5048 \times 10^3 + (100 + 24.2)^2 \times 6.293 \times 10^3 \right] + 2 \times 10 \times 500^3 \frac{1}{12}
\]

\[
= 412.5769 \times 10^6 \text{ mm}^4
\]

\( I_{yy} \) is min

\[
\delta_{min} = \sqrt{\frac{I_{yy}}{\text{Moment}}} = \sqrt{\frac{412.5769 \times 10^6}{2 \times (6.293 \times 10^3 + 5000)}}
\]

\[
= 135.155 \text{ mm}
\]

\( C_{st} = 500 \)
\[ n = \frac{A_0}{A_{\text{eff}}} = \frac{5000}{135.155} = 36.995 \]

\[ \sigma_{a_2} = 145 - \left( \frac{145 - 132}{20} \right) \times 14.393 \]

\[ = 140.453 \text{ N/mm}^2 \]

Safe load = 140.453 \times 2 \left( 6293 + 5000 \right)

\[ = 3172.277 \text{ KN} \]

If effective length is 6 m:

\[ n = \frac{6000}{135.155} = 44.393 \]

\[ \sigma_{a_2} = 145 - \left( \frac{145 - 132}{20} \right) \times 14.393 \]

\[ = 135.645 \text{ N/mm}^2 \]

Safe load = 135.645 \times 2 \left( 6293 + 5000 \right)

\[ = 3063.667 \text{ KN} \]
\[ d = \frac{\text{left}}{\gamma} = \frac{2550}{40.0 \times 27.5} \]

\[ A_{\text{one}} = \frac{20.0}{17.5} = 1.143 \]

\[ f = \frac{1220 - 1120 \times 9.257}{2.0} = 12.27 \text{ kN} \]

\[ F = 120 \times 0.288 \text{ N/mm}^2 \]
\[ f = 9.900 - \frac{900 - 7.20}{2.0} \times 2.727 \]

\[ = 875.45 \text{ MPa} = 85.882 \text{ N/mm}^2 \]

\[ P = 85.882 \times 2 \times 1379 = 236.863 \text{ KN} \]

\[ g = 9.81 \text{ N/s} \]

Note

As the angle is equal angle \( \tau_{xy} = \tau_{yy} \)
\( \tau_{xx} = \tau_{yy} \)

When put in combination \( z \) axis does not shift but \( y \) axis shifts such that \( \tau_{yy} \) of comb. > \( \tau_{xx} \) of comb.

hence
\( I_{\text{min}} \) of comb. = \( I_{xx} \) of comb.

Check for local buckling.

\[ \rho_{\text{we}} = \frac{30}{8 \nu} = \frac{30}{1.75} = 17.143 \]

\[ \rho_{\text{hole}} > \rho_{\text{we}} \]

\[ 17.14 \neq 92.7 \times 270.6^2 \]

\[ \neq 90 \]

\{ safe in local buckling. \}

Local buckling will only be checked load cannot be calculated on the basis of this.
A section of eff. length 6 m consists of twin box seen using an F5MB 250 with two plates 260 x 10 mm she welded each to the lips of two flanges of F5MB 250 on both sides with 4 mm fillet weld continuous through out the height properties of F5MB 250 are

\[ D = 250 \text{ mm} \]
\[ B = 125 \text{ mm} \]
\[ A = 40.47.55 \text{ cm}^2 \]
\[ I_{xx} = 5131.6 \text{ cm}^4 \]
\[ I_{yy} = 334.5 \text{ cm}^4 \]

Calculate the load carrying capacity of the section.

\[ I_{xx} = 5131.6 + 2 \times \frac{1 \times 26^3}{12} = 8060.933 \text{ cm}^4 \]
\[ I_{yy} = 334.5 \times 10^4 + 2 \left( \frac{260 \times 10^3}{12} + \frac{67.5 \times 260 \times 10}{12} \right) \]
\[ = 27.08 \times 10^6 \text{ mm}^4 \]
\[ J_{min} = 27.08 \times 10^6 \]
\[ \rho_{min} = \sqrt{\frac{J_{min}}{A}} = \sqrt{\frac{27.08 \times 10^6}{4355 + 260 \times 10 \times 2}} \]
\[ = 52.1567 \text{ mm} \]
\[ \frac{\rho_{eff}}{\rho_{min}} = \frac{6000}{52.1567} = 115.038 \]
Local buckling phenomenon in cover plates and web plates.

Local buckling phenomenon in webs and cover plate of composite member occurs in the form of waves or wrinkles. The critical stress at which local buckling in the form of waves or wrinkle starts is given by:

\[ f_{cr} = \frac{K \pi^2 E}{12(1-\nu^2) \left(\frac{b}{t}\right)^2} \]

\[ f_{cr} \propto \frac{1}{(b/t)^2} \]

Larger the value of \( b/t \), smaller is the stress at which local buckling starts in the form of waves or wrinkles. If \( (b/t) \) is so adjusted that critical stress for local buckling becomes more than the stress corresponding to overall failure of column, then local buckling will not occur before overall failure of column on this.
basis. IS code has given the following recommendations:

\[ \frac{(b/t)}{16} \neq 1 \]

Any portion of \( b \) greater than 16\( t \) should not be taken into account in strength calculation.

For web

\[ \frac{(d'/tw)}{50} \neq 1 \]

Any portion of \( d' > 50 \) \( tw \) should not be taken into account in strength calculation.

Note:
The area reduction should be done in such a way that it leads to maximum reduction in strength.
\[ I_{yy} = \frac{2 \times 198^3 \times 6}{12} + \frac{68 \times 300}{12} = 7.7677 \times 10^6 \text{mm}^4 \]

\[ I_{xx} = \frac{6 \times 300^3}{12} + 2 \left( \frac{198^3}{12} + 198 \times 6 \times 20^2 \right) \]

\[ = 111.42 \times 10^6 \text{mm}^4 \]

\[ \psi_{\text{min}} = \sqrt{\frac{I_{\text{min}}}{A}} = \sqrt{\frac{7.7677 \times 10^6}{2 \times 198 \times 6 \times 300 \times 6}} = 4.829 \text{mm} \]

\[ N = 81.15 \]

\[ \sigma_{xx} = 997.91 \text{ MPa} \]

\[ P_{\text{safe}} = 997.91 \times (2 \times 198 \times 6 + 300 \times 6) \times 10^2 \]

\[ = 41672.17 \text{ kg} \]

\[ = 41672.7 \text{ kN} \]
A companion is to be constructed using a BS 5370 for F500 from steel table = 23.6 mm

\[ \sigma_{ac} = \frac{f_c}{f_{0.2}} \]

\[ = \frac{3200/3}{1 + \frac{1}{7500} A^2} \]

\[ = \frac{800000}{7500 + A^2} = 75.9 \text{ MPa} \]

For max. efforce,

\[ f_{xx} = f_{yy} \]

\[ J_{xx} = 2 \times 6362.2 \times 10^4 \]

\[ J_{yy} = [310.84 \times 10^4 + 4564 \times (23.6 + \frac{3}{2})^2] \times 2 \]

\[ S = 183.99 \text{ mm}^2 \]

\[ \gamma_{min} = (\gamma_{xx})_{conh} = (\gamma_{yy})_{conh} = 118.1 \text{ mm}^2 \]

\[ h_{eff} = 6.5 \times 8 = 6.5m = 6500 \]

\[ d = 55.0 \text{ g} \]

\[ \sigma = \]
\[
\text{Psafe} = \sigma_{ae} \times \text{Area}
\]
\[
= 759.79 \times 4564 \times 2
\]
\[
= 69853.6 N \quad \text{kg}
\]

- \[
1 \text{ kips} = \frac{1 \text{ kN}}{2.2046 \text{ kN/m}^2} \quad \text{N/m}^2
\]

use this conversion

\[\text{f} = \text{force}\]

Design of compression member required

Design of compression member required finding out area of cross section which depend on \(\sigma_{ae}\) \((\text{area}_{fg} = \frac{P}{\sigma_{ae}})\).

But \(\sigma_{ae}\) also depends on the section hence we will not have a direct relationship as in the case of tension member where permissible stress was constant hence we will go for a trial and error solution.

**Step 1**

Assume the value of \(\sigma_{ae}\)

a) For rolled section choose \(\sigma_{ae}\) between 60—80 MPa.

b) For built up section choose \(\sigma_{ae}\) 110 MPa.

**Step 2**

\[
\frac{P}{(\sigma_{ae})\text{chosen}} = \text{Area required}
\]

Choose see trial see and find the slenderness ratio and hence \(\sigma_{ae}\) for the chosen area.
Calculate safe load carrying capacity of the chosen section

\[ p' = \text{area chosen} \times \sigma_{ae} \text{ chosen} \]
\[ p = \text{area provided} \times \sigma_{ae} \]

\[ p' \text{ safe load carrying capacity of trial secn} \]

- If \( p' > p \) secn is sufficient otherwise chose another larger section and repeat.

Check for max. limit of slenderness ratio.
Check for slenderness ratio can be done at the time when slenderness ratio of chosen secn was found out.

Design a built up column 10 m long to carry an axial load of 750 kn. Use 2 Channels placed back to back.

Load = 10 m. The section available are

**ISMC 250**

\[ A = 3867 \text{ mm}^2 \]
\[ t_{xx} = 99.4 \text{ mm} \]
\[ t_{yy} = 23.8 \text{ mm} \]
\[ C_{yy} = 21 \text{ mm} \]

**ISMC 300**

\[ A = 4564 \text{ mm}^2 \]
\[ t_{xx} = 118.1 \text{ mm} \]
\[ t_{yy} = 26.1 \text{ mm} \]
\[ C_{yy} = 23.6 \text{ mm} \]
1. chose \( \sigma_{aw} = 110 \text{ N/mm}^2 \)

\[ \Delta \text{req.} = \frac{P}{\sigma_{aw}} = \frac{750 \times 10^3}{110} = 6818 \text{ mm}^2 \]

chose 2 nos of LSI MC 256

A provided = \( 2 \times 384.7 = 7734 \text{ mm}^2 \)

For the chosen area find \( \delta_{\text{min}} \) and \( \delta \)

\[ \delta_{\text{min}} \leq \min \text{ of } \delta_x \text{ and } \delta_y \]

\[ \delta_x \text{ of combination } \]

\[ \delta_y \text{ of } \]

\[ \text{spacing between the seen for most efficient design} \]

\[ \delta_{xx \text{ comb}} = \delta_{yy \text{ comb}} \]

Thus spacing is so chosen that \( \delta_{xx} \text{ of comb } = \delta_{yy} \text{ of comm} \)

and under the condition

\[ \delta_{\text{min}} = (\delta_{xx} \text{ of comb}) \]

\[ \delta_{\text{min}} = \delta_{\text{xx} \text{ of one seen } = 99.4 \text{ mm}} \]

\[ \text{left} = 10 \text{ m} \]

\[ \text{side} = \frac{\text{left}}{\delta_{\text{min}}} = 100 - 6 < 180 \text{ ex. from the bond } \]
\[ N = 106.7 \]
\[ 60 = 92.8 \]
\[ 90 = 84 \]
\[ 100 = 75.3 \]

\[ \sigma_{ae} = 88.478 \text{ N/mm}^2 \]

\[ p^1 = \sigma_{ae} \times \text{Area of steel} \]
\[ = 84.478 \times (2 \times 3867) = 645.6 \text{ kN} < 750 \text{ kN safe} \]

Choose 2 I50MC 300

\[ A_{\text{provided}} = 2 \times 4564 = 9128 \text{ mm}^2 \]

\[ \sigma_{\text{min}} = \frac{F_0}{A_{\text{min}}} = 118.1 \text{ mm} \]

\[ d = \frac{I_{\text{eff}}}{A_{\text{min}}} = \frac{10.020}{118.1} = 84.67 < 180 \text{ ok} \]

\[ \sigma_{ae} = 100.7 - \frac{100.7 - 92.8 \times 24.67}{30} \]
\[ = 94.2 \text{ N/mm}^2 \]

\[ p^1 = \sigma_{ae} \times A_{\text{pro}} \]
\[ = 94.2 \times 9128 = 859.89 \text{ kN} > 750 \text{ kN ok} \]

Spacing calculation

\[ L_{xy} = \frac{L_{xy}}{c} \]
\[ 2 L_{xy} = 2 \left[ f_{xy} + A \left( c_{xy} + \frac{s}{2} \right)^2 \right] \]
\[ 2 A_{xy}^2 = 2 \left[ A_{xy} + A \left( c_{xy} + \frac{s}{2} \right) \right] \]
\[ \sigma_{xy}^2 = \sigma_{yy}^2 + (c_{yy} + \frac{e}{L})^2 + \frac{e}{L} \]

\[ S = 187.1 \text{ mm} \]

Spacing should be chosen 185 mm

Under this spacing, \( \sigma_{xy} \) will remain.

Design of lacing and batten

- **Single lacing**
  - (I)
  - (II)

- **Double lacing**
  - (III)

- **Batten**
  - (IV)
1) Out of (1), (2) and (3), (3) is better because accidental failure of (2) joint will not decrease the distance b/s. Intermediate connection significantly.

2) If the single lacing is not sufficient to safeguard against local buckling, double lacing will be provided.

Lacing

Lacing is in the form of a flat plate or angle. The y/y is kept slightly more than x/x.

Lacing should not be varied throughout the length of compression member (angle, of x-section, etc. should be same).

Single lacing on opposite side of main component should be mirror image of each other.

Lacing on adjacent face should be staggered.

All the top and bottom in the laced column batten plates are provided they are called end battens.

1-2 mirror image
3-4 mirror image
1-3 staggered.
Design

1) Angle of inclination of lacing with the longitudinal axis of column should be between 90 - 70 degrees.

2) Slenderness ratio of individual member of column b/w intermediate connections should not be more than 50 either more than 0.7 times slenderness ratio of whole section.

\[
\frac{L}{r_{min}} \neq 50 \\
\frac{L}{r_{0.7 \text{ whole}}} \neq 0.7
\]

Note: To start with, an angle will be chosen. As spacing has already been chosen for component members, \( c \) becomes fixed. This can be checked for local buckling criteria.

If it is not met, \( \theta \) can be increased.

Effective length of lacing bar

\[
\ell_{eff} = l \text{ for single lacing (riveted)} \\
\ell_{eff} = 0.7l \text{ for double lacing} \\
\ell_{eff} = 0.7l \text{ for welded lacing (when there is connection at the overlap)}
\]

\( l \rightarrow \) distance b/w inner ends of rivet or weld

Slenderness ratio of lacing bar should not be more than 145

\[
\frac{\ell_{eff}}{r_{min}} \neq 145
\]
\[ b \]  

\[ V_{\text{min}} = V_{yy} = \sqrt{\frac{f_{yy}}{A}} = \sqrt{\frac{bt^3}{5t^2}} = \frac{t}{\sqrt{12}} \]

\[ \frac{bf}{t/\sqrt{12}} = 17.5 \]

Min. width of lacing bar

Nominal dia of rivet (mm):

22  
20  
18  
16

Min. width of lacing bar (mm):

6.5  
6.0  
5.5  
5.0

Min. thickness of lacing bar:

Min. thickness \( \geq \frac{t}{40} \) for single lacing  
\( \frac{1}{60} \) for double lacing

Force for which lacing should be designed:

The lacing should be designed for shear force in column arising due to:

a) 0.m.  

b) Lateral loading  

c) Incidental Eccentricity  

\( \Rightarrow \) Unavoidable
For incidental eccentricity transverse shear = 0.5% of axial load in the member is taken into account.

This shear is divided equally between all transverse lacing system in parallel planes.

\[
\begin{align*}
\text{Shear} &= \frac{V}{L} \\
\text{Lacing bar will be designed for tension and compression the force will be taken equal to} \left(\frac{V}{2 \sin \theta}\right) \\
\text{Check for tension} & \quad (b-d') t \sigma_{at} = \frac{V}{2 \sin \theta} \\
\text{Check for compression} & \quad \frac{V}{2 \sin \theta} < \sigma_c \quad \text{(corresponding to effective strain } \frac{1}{E})
\end{align*}
\]
Design for connection of lacing bar

$$\frac{V}{2 \sin \theta} \leq R_v$$

Single shear

$$\frac{V}{2} \sin \theta$$

Note:

Shear at sec 1 will correspond to $$\frac{V}{2 \sin \theta}$$

Shear at sec 2: $$V \cot \theta$$

Normally check is done for $$V \cot \theta$$. However, if $$\frac{V}{2 \sin \theta}$$ becomes more than $$V \cot \theta$$, means $$\frac{V}{2 \sin \theta} > V \cot \theta$$ \Rightarrow \theta > 60^\circ$$

Sec 1 will become critical here.
check should be done for \( \frac{V}{2\sin\theta} \) force but normally \( \theta \) is around 45°, so \( (V\cos\theta) \) governs.

\[ P = 250 \text{ kN} \]
\[ A = 4 \times 19.03 = 76.12 \text{ mm}^2 \]
\[ \sigma_{xy} = \frac{750 \times 10^3}{76.12} = 98.13 \text{ N/mm}^2 \]
\[ \chi_{yf} = 82.247 \]
\[ \chi_{yf} < 82.247 \text{ mm} \]
\[ \frac{\sigma_{eff}}{\sigma_{min}} < 0.2 \]
\[ r_{min} > 114.29 \text{ mm} \]

\[ J = A \times r^2 \]
\[ = (4 \times 19.03) \times 114.29^2 = 99.43 \times 10^6 \text{ mm}^4 \]

\[ J_{min} = 9 \times \left[ 1.77 \times 10^4 + \left( (100 - 28.4) + \frac{5}{2} \times 19.03 \right)^2 \right] = 99.43 \times 10^6 \]

\[ S = 77.09 \text{ mm} \]

Adopt \( S = 80 \text{ mm} \)
taking $g = 40 \text{ mm}$

\[ c = \frac{2a_0}{c/2} \]

\[ \tan \theta = \frac{a_0}{c} \]

Assuming $\theta = 45^\circ$

\[ c = 400 \text{ mm} \]

Check for local buckling

\[ \frac{c}{f_{\text{min}}^\text{ind}} \neq 50 \]

\[ \frac{400}{f_{\text{yy}}} \neq 0.7 \text{ kN/m} \]

\[ \frac{400}{19.4} = 20.62 \neq 50 \quad \text{OK} \]
Note: for slenderness ratio \(83.247\) spacing is 73.09.

If spacing is increased, \(\delta_{min}\) will increase hence \(I\) will fall thus for spacing equal to \(I\) would be less than 83.247. If local buckling criteria is satisfied for longer slenderness ratio.

Design of lacing

Local buckling criteria is satisfied for \(\theta = 45\) in single lacing. Hence single lacing is sufficient.

\[
l = 200 \sqrt{2} = 282.843 \text{ mm}
\]

\[
\delta_{min} = 6.05 \sqrt{I} = 6.05 \sqrt{10} = 17.11 \text{ mm}
\]

Min. width of lacing = 60 mm

\[
\text{Min. thickness } = \frac{l}{40} = \frac{282.843}{40} = 7.07 \text{ mm}
\]

Check for slenderness ratio of lacing

\[
\frac{I_{eff}}{t \sqrt{I}} \neq 145
\]

\[
\frac{282.843 \sqrt{I}}{8} \neq 145
\]

122.47 \(\neq\) 145

OK.
Check for tension:

\[ V = 5.5\% \text{ of } P \]

\[
\frac{2.5 \times 7.50}{100} = 18.75\text{ KN}
\]

Tensile force in lacings:

\[ f = \frac{V}{\sin\theta} = 13.25\text{ KN} \]

Tearing strength:

\[ (60 - 21.5) \times 8 \times 0.6 f_y > 13.25 \]

\[ 46.2\text{ KN} > 13.25\text{ KN} \]

Check for compression:

\[ P = 122.5\text{ KN} \]

\[ \sigma_{ac} = 65.27\text{ N/mm}^2 \]

\[ P' = \sigma_{ac} b d \]

\[ = 65.27 \times 60 \times 8 = 31.32\text{ KN} > 13.25\text{ KN} \]

Safe in compression.

Design for connection:

\[ V_{col} = 18.75\text{ KN} \]
Rivet value = \( 3 \) \( \text{Min (shearing, bearing)} \)

Adopting torsion

\[ \text{tension shop rivet} = \min \left\{ \frac{1241}{E}, \frac{300 \times 21.5 \times 18}{51.6} \right\} \]

Shearing strength of rivet = 36.3 kN

No. of rivet = \( \frac{18.75}{36.3} \)

Adopt 1 rivet

Check for bearing

\[ \rho = 12.25 \text{kN} \]

64.5 \( \geq \) 18.75 \ OK.

51.6 \( \geq \) 13.25 \ OK

\[ \{ \text{safe in bearing} \} \]

Design of end batten

\[ \text{b} = 100 \text{mm} \]

\[ d = \text{eff. depth of end batten} \]

\[ d \neq a \neq 223.2 \] \( \Rightarrow \) Adopt \( d = 225 \text{ mm} \)

\[ d \neq 2b \neq 2a \] \( \Rightarrow \) Adopt \( d = 225 \text{ mm} \)

\[ d + d + 2 \text{ edge distance} = 225 + (2 \times 2a) \times 2 = 305 \text{ mm} \]
Thicknss \(= \frac{9}{50} = \frac{280}{50} = 4\)

Adhesive 8 mm
Design of End Batten.

The effective length of batten column is taken 10% more than that of faced column.

The $\gamma_{yy} > \gamma_{xx}$

$\gamma_{yy}$ plane is the plane of to the plane of batten.

Batten on opposite faces should be mirror image. No of batten should be such that it divides the column longitudinally in not less than 3 parts.

Effective length of battened column is taken 10% more.

Design Specification:

$$\frac{c}{(T_{min})_{b,c}} \times 50$$

$\phi = 0.7$ for stability.
Effective depth of intermediate batten should not be less than \( \frac{3}{4} \) times the distance b/w centroid of component members. The eff. depth however should not be less than twice the width of 1 component member in the plane of batten.

\[ d = s + 2c_{yy} \]

\[ d' \neq \frac{3}{4} d' \]

\[ \neq 2b \]

For End batten:

Effective depth of end batten \( d' \neq d' \neq 2b \)

The thickness of batten should not be less than \( \frac{l}{50} \)

where \( l \) is the distance b/w innermost connecting line of rivet or weld.

\[ l = (s + 2y) \]
Check for safety

Batten should be designed to carry B.M. and shear arising due to transverse shear. Transverse shear for incidental eccentricity:

\[ v = \frac{2.5}{100} \text{ Poutl} \]

Hence batten plate will be designed for

Shear: \[ \frac{vc}{2l} \]
Moment: \[ \frac{vc}{4} \]

Check for shear

Permissible average shear stress = 0.4 fy
\[ \frac{(\frac{vc}{2l})}{bt} \leq 0.4 \text{ fy} \]

Check for bending

Permissible average bending stress = 185 N/mm²
\[ \left( \frac{\frac{vc}{4} \times D^2}{t \times D^3} \right) \leq 185 \text{ N/mm²} \text{ or } 165 \text{ N/mm²} \]
Design of connection will be done as eccentric connection in which rivet group is subjected to transverse shear of \( \frac{Ve}{2t} \) and torsional moment \( \frac{Ve}{t} \).

From the torsional moment (or torsional shear) and transverse shear, shear force in extreme rivet will be found out and this should be less than rivet value for safety of connection.

Welded Connection:

\[ \text{Overlap} \neq 4t \]

\[ t \rightarrow \text{thickness of plate} \]

Total length of weld at the edge of batten \( \neq \frac{D}{2} \)

(P.1 + I.2 + I.4) \( \neq \frac{D}{2} \)

Practically we provide continuous connection hence their recommendation will not be required to be checked.
Column splices

Case (a)

Top storey column & bottom storey column are of same size

Case (b)

top & bottom storey column are of different size but top storey column flange resting completely on bottom storey flange
In case of complete bearing of top storey column flange on bottom storey column flange the design is as follows:

If the column ends are milled all of the direct force will be assumed to be transferred through direct bearing of top storey to bottom storey. (IS code)

If moments are not acting, the purpose of splice plate will only be to hold the two columns together.

However, to find out the size of splice plate it will be assumed that 50% of the direct load is transferred through splice plates.

Hence force on one splice plate = \( \frac{F}{4} \)

Area of splice plate = \( \frac{F}{4} \cdot \frac{150}{60} \)

As there is no buckling i.e., \( \sigma_{ax} = 0.6f_y \)

\[ b \times t = \frac{(9/4)}{0.6f_y} \]

- If the column ends are not milled design force = \( \frac{1}{2} \) for one splice

If the column is subjected to direct load as well as moment:

**Case 1:** When column ends are milled

[Additional content not visible in the image]
All direct force will be assumed to transferred through direct bearing. Splice plate however will be designed for forces generated due to moment. Hence design force = \( \frac{M}{l} \)

\( l \rightarrow \) spacing b/w splice plate

(Case 1)

When column ends are not milled

Design force \( \text{in compression} = \left( \frac{p}{2} + \frac{M}{l} \right) \)

Design force \( \text{in tension} = \left( \frac{p}{2} - \frac{M}{l} \right) \)

\( l \) becomes \( \text{when} \) value becomes 0, \( v \)

\( \frac{V}{2(a+t)} \leq 0.4 f_y \)

Web splice are provided to resist shear in the column

Max space available for providing splice plate is

\( (D - 2h_g) \)

\( a \) should be chosen smaller than this

Rivets here will be designed to be in double shear.
When top story column flange is not resting on the bottom story column flange.

In this case the total top load is assumed to have been transferred through the column flange to the base plate and then through the base plate to the bottom story column flanges. The connection will be designed by usual methods. However, the base plate needs to be designed.

\[ D_t = \text{Depth of top story column} \]
\[ T_b = \text{Thickness of base plate} \]

\[ (D_t - T_d) = b \]

\[ D_b - T_b = a \]

\[ M_{\text{max}} = \frac{P(a - b)}{4} \]

For bending stress, \[ \sigma = \frac{F_{\text{max}} \times Z}{l} = \frac{P(a - b)}{4} \]

165 for flanged
185 for solid

M.O.R.
Open \( \frac{bt^2}{6} = \frac{p(a-b)}{t} \) from here calc. \( t \)

Note:

1) If packing is required, thickness of packing will be equal to the gap to be filled.

2) In this case if thickness of packing is greater than 6mm, additional rivets will be provided on packing extension and no. of such rivets is equal to 2.5% of the no. of rivets obtained from normal calculation per 2 mm thickness of packing.

- In case of partial bearing all the direct load will be assumed to have been transferred from top story column to bottom story column through splice plate.

Question
2 ISMB 300 sections are spliced using compression splice the applied load is 300 kN, S.F. is 75 kN and applied moment is 20 kNm assume column ends to be not milled. Design column splice

As the column ends are not milled all of the direct force will be transferred through the splice plate.

Hence force in splice plate = \( \frac{P}{2} + \frac{M}{l} \)

Adopt \( l = 300 \text{ mm} \)

to be on safer side

The forces in splice are

\[
\begin{align*}
\frac{P}{2} + \frac{M}{l} &= 216.07 \text{ kN comp.} \\
\frac{P}{2} - \frac{M}{l} &= 83.33 \text{ kN comp.}
\end{align*}
\]
Design force for column splice = 216.67 kN
Permissible stress in compression in splice = 0.6f_y

 splice will not buckle individually

Area required = \( \frac{216.67 \times 10^3}{0.6 \times 2.50} \)

\[ = 1444.47 \text{ mm}^2 \]

Width of splice plate is taken as the width of column flange

\[ b \times t = 1444.47 \]
\[ b = 250 \]
\[ \text{and } t = \frac{1444.47}{250} = 5.7 \]

Adopt \( t = 8 \text{ mm} \)

Length of column splice will depend on no. of rivets

Design of connection

Rivets will be designed for force of 216.67 kN

Adopting the dia. of rivet as 18 mm and choosing power driven field driven

For stress in rivet in shear = 90 MPa

\[ \text{bearing} = 270 \text{ MPa} \]

Rivet value \( \text{min} \left\{ \text{shear, bearing} \right\} \)

\[ = \frac{4}{\pi} \times 18.125^{0.99} = 19.5 \times 8 \times 170 \]

\[ \text{min} \left\{ \text{26.87 kN} \right\} = 42.12 \text{ kN} \]
$N = 26.87$ KN

No of level $= \frac{218.67}{26.87} = 8.06$

Adopt 10 No. (even no. due to symmetry)

\[ f = 8f + 4c = 8 \times 2.5d + 4 \times 2d' = 28d - 18 \times 18 = 50.4 \text{ mm} \]

\[ = 20 \times d + 8d' = 520 \text{ mm} \]

Splice plate on the other side will also be of same

\[ \text{area} = \text{connection detailing will also be same.} \]

Design for shear splice:

Area required to resist shear

\[ = \frac{75}{0.4fy} \]

\[ = \frac{75 \times 10^3}{100} = 750 \text{ mm}^2 \]

\[ 2at \geq 750 \]

Adopt width of splice $12.5 \text{ mm}$

\[ t_{sp} = \frac{750}{12.5} = 60 \text{ mm} \]

\[ \geq 3 \text{ mm} \]

Adopt 3mm thickness of plate $= 6 \text{ mm}$
rivets are in double shear hence

\[ R_v = \min \left\{ \frac{2 \pi R^2}{4} \times 90, \frac{6 \times 90}{7.6} \right\} \]

\[ = \min \left\{ 53.75, 40.014 \text{ KN} \right\} \]

\[ R_v = 40.014 \text{ KN} \]

No. of rivet \[ \frac{75}{40.014} \approx 1.8 \]

adopt 2 rivet

Note:

\[ \text{ISSC} \rightarrow \text{Is a } \# \text{ I section} \]

Q. An I5HBB 300 in lower story and I5HBB 200 in the upper story has been used

I5HBB 300
D = 300
B = 250
T = 10.6
\[ t_o = 7.6 \]

Column load is 650 KN design the base plate placed between top & bottom story column.

For the top story column flange is not resting at all on the bottom story column flange hence we require the base plate.

Size of the base plate will be \[ 300 \times 250 \times 1 \]
\[ \frac{P}{(a-b)} \leq \text{M.O.R.} \]

\[ \frac{10^{-3} \times 650 \times (289.4 - 191)}{4} \leq \sigma_{\text{Bcr}} \cdot 2 \]

\[ \leq \frac{0.66 \times 150 \times 250 \times t^2}{185} \cdot 6 \]

\[ t \geq 45.54 \text{ mm} \]

Adopting \( t = 50 \text{ mm} \)

Adopting 0.185 N/mm² for solid plate

# Column Bases
If steel column is directly placed on a concrete pedestal, the concrete will get crushed. To safeguard against crushing of concrete area, the load transferred should be increased so that the resulting stress on concrete is less than the permissible stress in direct compression in concrete.

The various types of column bases are:

a) Slab Base

b) Grussetted Base

c) Grillage Foundation

![Diagram of column base and grillage foundation]

Cleat Angle
ISA 60x60x8

(slub base)

(grussetted base)
Grillage Foundation.

Note: Grouted base is better if the column loads are heavy.

It provides greater rigidity at the joint.

When the load is heavy and soil is weak or when the foundation is to be laid at shallow depth we provide

Grillage foundation. Grillage foundation provides larger area at the base hence the pressure on the soil reduces.

Grillage foundation is also provided in case of temporary construction to temporarily support the structure.

When the column load is purely axial.

Assuming that column transfers a load of $P$ at the base plate

Size of base plate will be decided as

$$\frac{P}{\sigma_c} = \text{Area of base plate}$$
Note - If applied load is given we should take into account the dead wt. also hence the design force for base plate will be 1.1 × P where P = applied load on column and 1.1 takes into account the dead wt. which is taken as 10% of applied load.

For purely axially loaded column

\[
\begin{align*}
 t &= \sqrt{\frac{3w}{a_{th}} \left( \frac{a^2 - b^2}{4} \right)} \quad \text{a larger overhang} \\
 t &= \sqrt{\frac{3w}{a_{th}} \left( \frac{a^2 - b^2}{4} \right)} \quad \text{b smaller overhang}
\end{align*}
\]

\( t \) - Thickness of base plate

\( \sigma_{th} \) - Pres. bending stress in base plate (185 N/mm²)

\( w \) - load on base plate

Area of base plate

\[
\begin{align*}
 M_{11} &= \frac{wa^2}{2} \\
 M_{22} &= \frac{wb^2}{2} \\
 \sigma_{11} &= \frac{M_{11} \times t}{I_x} \\
 \sigma_{22} &= \frac{M_{22} t/2}{I_x \times t^3/12}
\end{align*}
\]
\[
\frac{\Delta A}{E} = \frac{\sigma_{II} \delta}{E} - \frac{\mu \sigma_{III}}{E} = \left( M_{II} - \mu M_{III} \right) \times 6 = \left( \frac{3\omega^2 - \frac{11\omega b^2}{2}}{2} \right) \frac{1}{t^2}
\]

\[
\sigma_A = \frac{3\omega \left( a^2 - ab \right)}{t^2} \Rightarrow t \geq \sqrt{\frac{3\omega}{\delta_{bs}} \left( a^2 - ab \right)}
\]

Assuming \( \mu = 0.35 \) for steel

\[
t_{\min} = \sqrt{\frac{3\omega}{\delta_{bs}} \left( a^2 - \frac{b^2}{4} \right)}
\]

Note: The plate is critical in bending at section 1-1 hence thickness is decided on the basis of bending however their for shear can be done as

\[
\frac{\omega x 1 x a}{1 x t} \leq 0.4 f_y
\]

If the column is subjected to load as well as moment

Net base for is to be calculated and bending stress at critical section is evaluated. This bending stress is shown to be less than the permissible bending stress. This analysis helps us in finding out thickness of base plate.
Slab base subjected to moment and axial loading:

\[ \frac{P}{BD} \left( 1 - \frac{6e}{D} \right) \]

Effect of axial loading:
\[ \frac{P}{BD} \]

Effect of bending:
\[ \frac{M}{I} \]

Net stress:
\[ \frac{P}{BD} + \frac{6Pe}{BD^2} \]

If \( c < \frac{D}{6} \)
\[ \frac{6e}{D} < 1 \]

For design:
\[ \frac{P}{BD} \left( 1 - \frac{6e}{D} \right) \leq \sigma_c \]

Load per stress in cm.
Using this formula we can choose $t$ and find out other (choose either of $B$ and $D$ and calculate other).

Thus, size of base plate will be known -

Thickness of base plate

Section 1-1 is the critical section for bending.

\[
\sigma_A = \sigma_{11} - \mu \sigma_{22} = \frac{M_{22} t}{2} - \frac{M_{11} t/2}{t^2/12} = \frac{M_{22} t}{12 + 3t/12} \\
\sigma_A = \frac{6}{t^2} \left( M_{11} - \mu M_{22} \right) \leq \sigma_{bs}
\]
If value of \( \mu \) is not given we can neglect it by doing so we will be on safer side. 

\[
\sigma = \sqrt{6 \left( \frac{\sigma_{\text{m}} - \mu \sigma_{\text{p}}}{6} \right)}
\]

Max. Stress: 
\[
\sigma_{\text{max}} = \frac{P}{BD} \left( 1 + \frac{6e}{D} \right)
\]

Min. Stress: 
\[
\sigma_{\text{min}} = \frac{P}{BD} \left( 1 - \frac{6e}{D} \right)
\]

If \( e > \frac{D}{6} \), 
\[
\sigma_{\text{min}} = \frac{P}{BD} \left( 1 - \frac{6e}{D} \right) \text{ becomes } 0 \text{ we call this}
\]

given to tensile stress there will be loss of contact if you have plate and concrete, In the final length of contact situation minimum stress will be zero. For this

\[3 \left( \frac{d}{2} - e \right) = D'' \]
Hence final length of contact = \[ 3 \left( \frac{D}{2} - e \right) \]

\[ e = \frac{M - P}{P} \]

and Max. comp. stress = \[ \frac{2P}{3\beta \left( \frac{D}{2} - e \right)} = \frac{2P}{3\beta \left( \frac{D}{2} - e \right)} \]

\[ e > \frac{D}{6} \]

In the starting \( e \) will be known \( \left( \frac{M^2}{P} \right) \) Hence \( D \) will be chosen such that \( e < \frac{D}{6} \). In that situation stress will be compressive through out and design will be done as discussed in previous case.

However if \( D \) is given, then if \( e \geq \frac{D}{6} \), there would be loss of contact. Hence final length of contact and maxm compersive stress will be calculated.

From this stress distribution maxm bending stress at critical section 1-1 will be calculated and should be \( \leq \) per bending stress

\[ \sigma_c = 3.75 \text{ MPa} \]

\[ f_y = 250 \text{ MPa} \]

\[ T = 15 \text{ mm} \]

\[ f_{st} = 0.7 f_y = 175 \text{ N/mm}^2 \text{ per mm} \text{ strain in bending tension} \]
Shearing force that can be resisted by \( \sigma_c \): 

\[ \sigma_c = \frac{M}{P} = \frac{5.5}{700} = 7.857 \text{ mm} \]

Choose \( D > 6 \sigma_c \) 

\( 471.43 \text{ mm} \)

Choose \( D = 500 \text{ mm} \)

Max. stress 

\[ \frac{P}{BD} \left( 1 + \frac{6\sigma_c}{D} \right) \leq \sigma_c \]

\[ \frac{700}{8 \times 500} \left( 1 + \frac{6 \times 7.857}{500} \right) \leq 3.75 \]

\( R > 725.37 \text{ mm} \)

Adopt \( B = 730 \text{ mm} \)

Note: by increasing the value of \( D \) overhang portion will increase but stress will decrease. However, by choosing smaller \( D \) overhang will reduce, but stress will increase. Final selection is generally done based on economy, i.e., cost of steel \( B \times D \times t \) should be less.
Stress 

\[ \sigma = \frac{P}{BD} \left( 1 - \frac{a}{D} \right) \] 

\[ \frac{10^9 N}{mm^2} \]

Note if connection has been made using angles the length of plate should be chosen such that the connecting angle could be accommodated in the chosen length.

\[ \sigma_n = \frac{M_{n_1} t^{1/2}}{I_2^{1/2}} = \frac{EI M_{2_2} t^{1/2}}{I_1 t^{1/2}} \leq \sigma_k \]

\[ 2.641 \]

\[ 79.26 \]

Force on 1 mm width = \[ \frac{1}{2} \left( 2.641 + 8.726 \right) \times 150 \times 1 \]

\[ \frac{777.45 N}{mm} \]

\[ \frac{477.45 \times 79.26 = 37848.275 N/mm}{L_{2_2}} = \frac{2.641 \times 265^2}{2} = 92728.6 N/mm \]
\[
\frac{6}{t^2} \left( M_{nt} - u M_{nt} \right) \leq 0.7 \times 1250
\]
\[
t \geq 34.135 \text{ mm}
\]

Similarly:
\[
\frac{6}{t^2} \left( M_{nt} - u M_{nt} \right) \leq 0.7 \times 1250
\]
\[
t \geq 53.43 \text{ mm}
\]

Let us adopt max. thickness:
\[
t = 55 \text{ mm}
\]

\[B = 730 \text{ mm} \]
\[d = 500 \text{ mm} \]
\[t = 55 \text{ mm} \]

**Design of weld.**

As the column ends are milled, all the direct load will be assumed to have been transferred through direct bearing on the base plate hence weld should be designed only to resist the moment.

\[
\frac{M}{d} = \frac{55 \times 10^6}{200} = 275 \text{ MN}
\]

\[d \times 5 \times 76 \geq 275 \times 10^3 \]

Min. size = 10 mm

Max. size = 15 - 1.5 = 13.5 mm max.
Assume $l = 370$ mm

10 mm fillet weld

Note: If there is tension at the base, the bolt will come under tension and we need to design the bolt as well. Bolt can be designed if the tension in the bolt can be estimated. Actual determination of $T$ will be based on the following fig.

However, a conservative estimate of tension in the bolt is as follows.
Once tension is known the dia of bolt and length of embedment can be calculated.

\[ \pi \phi \times \text{Lemb} \times \sigma_a \geq T \quad (1) \]

Per. adhesive stress b/w bolt & con.
depends on grade of steel

\[ \frac{\pi \phi^2 \cdot \sigma_e}{\phi} \geq T \quad (ii) \]

\( \phi \) = A Dia at the root of thread

From (i) & (ii) dia of bolt & length of embedment can be calculated.

If a solid round steel column is supported over a square base plate

then for purely axial load thickness \( \ell \) is given by

\[ \ell = \frac{90W1}{16 \sigma_{fs}} \left( \frac{8}{8 - d_s} \right) \]
\[ W \rightarrow KN \]
\[ t \rightarrow mm \]
\[ \sigma_{th} \rightarrow MPa \]

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Design of Beam

For a beam to be safe

a) It should be safe in bending (primary criteria)
   safe in shear
   deflection

c)

d) safe in local buckling (secondary criteria)
of flange plate

e) safe in web crippling

f) safe in web buckling

Lateral restraint

Laterally restrained and unrestrained beams

Beam can buckle laterally if the compression flange is weak; hence, to safeguard against lateral buckling

a) Beam can be laterally restrained by using cross beams

```
+-----+-----+-----+
|     |     |     |
+-----+-----+-----+
```

- Main beam

```
+-----+-----+-----+
|     |     |     |
+-----+-----+-----+
```

- Cross beam

b) By inserting the compression flange inside the floor-girder slab
e) by making compression flange heavy.

For laterally restrained beam

\[ \delta_{bc} = \delta_{bt} \]

For stress in bending, tension = 0.67f_y

\[ = 165 \text{ MPa} \]

For stress in bending comp.

For laterally unrestrained beam \( \sigma_{bc} < \sigma_{bl} \)

Design of beam (laterally restrained beam)

1. Beam is designed for bending and checked for other criteria.

\[ \frac{M_{\text{max}}}{\sigma_{bc} \text{ or } \sigma_{bl}} = Z_{\text{ref}} \]

Choose section that provides \( Z > Z_{\text{ref}} \).

When \( Z_{\text{ref}} \) can not be provided by using single rolled section, we will adopt built up section.

Note

Most suitable arrangement for deflection criteria.

One of the most common built up section is I beam with flange plates.
In seeking and flange plate combination the largest I seek available is used and deficiency in $z$ is met by using plates.

$I_{eq} = I_{rolled} + I_{plates}$

$I_{req.} = \frac{I_{rolled}}{D^2} + \frac{I_{plates}}{D^2}$

$Z_{req.} = Z_{rolled} + A_p \times \left(\frac{D/2}{D}\right)^2 \times 2$ \[ \frac{Z_{req.} - Z_{rolled}}{D} = A_p \]

The above approach gives the approximate value of $A_p$ the area of plate chosen is more than $A_p$.

The increase in $A_p$ should reckon for the area of hole on the tension flange because in tension net area is effective.

The beam should be checked for safety in bending as follows:

\[ \frac{M \cdot \text{Y cmb}}{I_{greq}} \leq \sigma_{El} \]

\[ \frac{M \cdot \text{Y ten}}{I_{grea}} \left(\frac{\text{Gross area of tension flange}}{\text{Net area of tension flange}}\right) \leq \sigma_{Et} \]

$I_{grea}$ corresponds to $NA$ of section neglecting holes.
Check for shear

\[ \frac{V_{\text{max}}}{D + 2t} \leq 0.4 f_y \]

\[ V_{\text{max}} \rightarrow \text{Max. s.f. in Beam} \]

\[ \frac{V_{\text{max}}}{b \times t_0} = \text{Av. Shear Stress} \leq 0.4 f_y \]

\[ \text{per av. shear stress} \]

Note

\[ \text{Max. shear stress} \leq 0.45 f_y \]

\[ \text{Max. shear stress} \leq 0.95 f_y \]

\[ \frac{V_{r_{\text{max}}}}{(D - 2h_3)} \leq \text{Max. shear stress} \leq 0.9 f_y \]
Check for Deflection

Max. Permissible deflection $= \frac{\text{span}}{325}$

$$\Delta_{\text{max}} = \frac{s \cdot \text{w}_f \cdot t}{384 \cdot E_f} \leq \frac{1}{325}.$$

Check for secondary criteria

Local buckling of flange plate

Note: Secondary criteria need not to be checked for rolled section because sections are designed in such a way that overall failure of the section takes place before the local failure has had the chance to take place.

\[ \frac{a}{t} \neq 50 \]
\[ \frac{b}{t} \neq 50 \]

Plates are simply supported and placed one over the other. If however, two plates are welded so that they behave as 1 unit:

\[ \frac{b}{t} \neq 16 \]
This criteria should be checked at the time of selection of plate only.

Check for web crippling.

Web crippling takes place at the location of heavy point load. The heavy point load can come in the span and at the support. The point load are dispersed as shown below.

For no web crippling

\[ \frac{P}{\alpha t w} \leq \sigma_{b1} \quad \text{(per bearing stress)} \]

Web crippling takes place at location where section provides least area of resistance such location is first encountered at the root of fillet. Hence web crippling takes place at the root of fillet.

For safety against web crippling bearing stress at the root of fillet should be less than permissible bearing stress (0.75 fy)
At support:

\[ \frac{P}{[C \pm \frac{1}{2}(a + b)]} \leq \sigma_{bl} \]

Note: If the beam is carrying UDL, web crippling needs to be checked at the support.

The support width is decided based on the bearing failure of a supporting structure as well as web crippling of steel section.

Web Buckling:

Web buckling occurs due to heavy concentrated diagonal compression due to shear. To check for safety in web buckling.

\[ P_{1} \]

\[ P_{2} \]

\[ P_{1} \text{ (i)} \]

\[ P_{2} \text{ (ii)} \]

Area at N.A., \( y_{tw} \)
To check for web buckling web is treated as a column

\[ \text{Mendelsohn ratio} = \frac{d_w \sqrt{\frac{d}{L_2}}}{t_w} \quad \text{where} \]
\[ d_w = d - 2h \]
\[ t_w = \text{thickness of web} \]

The area resisting compression is calculated as follows (Fig ii)

For safety against web crippling

\[ \text{Carrying capacity} = \sigma_{ax} \times A > P \]
\[ \sigma_{ax} = \text{axial stress in axial compression obtained from} \]
\[ \text{Mendelsohn ratio} = \frac{d_w \sqrt{\frac{d}{L_2}}}{t_w} \]

\( P \implies \text{Applied load} \)

Note

Cote: The web can be treated as a column which is fixed at the two flange ends hence

\[ L_{eff} = \frac{d_w}{l} \]
\[ \tau_{min} = \sqrt{\frac{1}{A_{min}}} \]

\[ L_{eff} = \left( \frac{d_w}{l} \right) \frac{1}{tw} \]
\[ \tau_{min} = \sqrt{\frac{a + 3}{12 \times atw}} \]

\[ L_{eff} = \frac{d_w}{tw} \sqrt{\frac{d}{L_2}} \]
Generally if beam is safe in web wrinkling it will be safe in web buckling.

Design of rivets

No. of rivets per pitch

\[
\frac{V_{AY}}{P} = \text{Shear force per pitch length} \leq \frac{n}{Bv}
\]

where: 
- \(V_{AY}\): Shear force
- \(n\): No. of rivets
- \(B\): Pitch length
- \(L\): For safety

Rivet diameter will be assumed and pitch will be calculated for max. shear.

Same pitch can be adopted throughout the span.
In reality, as shear force decreases towards the mid span, the pitch should increase towards the mid span.

Curtailment of Plate

Theoretical cut-off point is the location at which B.M. is equal to M.O.R. of continuing section.

B.M. at $x_3$ location

$$\frac{wL}{2} \left( \frac{L}{2} - x_3 \right) - \frac{w}{2} \left( \frac{L}{2} - x_3 \right)^2$$

$$= \frac{wL}{2} \left[ \frac{L^2}{4} - \frac{Lx_3}{2} - \frac{x_3^2}{4} - x_3^2 \right]$$

$$= \frac{wL}{2} \left( \frac{L^2}{4} - x_3^2 \right)$$

$$\frac{wL}{2} \left( \frac{L^2}{4} - x_3^2 \right) = 0$$

find $x_3$

At least one plate must continue through out the span.
No rolled section can provide \( Z = Z_{\text{req}} \). Hence plates will be used to increase the \( Z \) value. Hence let us use 5BM 400 and cover plate as shown.

Approx area of plate req. \( \frac{Z_{\text{req}}}{\text{rolled}} = \frac{227.813 \times 10^6}{165} = 1.38 \times 10^4 \text{ mm}^2 \)

Let us choose \( A_p = 1200 \text{ mm}^2 \)

Adopting \( t = 8 \text{ mm} \)

Width req. \( \frac{1200}{8} = 150 \text{ mm} \)

Check for bending:

Area of tension flange = Area of flange plate + Area of flange of rolled sec.
As the thickness of flange of rolled 

\[
\text{area} = \frac{150 \times 8}{150 \times 8 - 21.5 \times 8 \times 2} = 1.4
\]

\[
I_{\text{gross}} = 205 \times 10^6 + 2 \left[ \frac{150 \times 8^3}{12} + 150 \times 8 \times 20.9^2 \right] = 304.89 \times 10^6 \text{ mm}^4
\]

\[
y_{\text{max}} = 208 \text{ mm}
\]

\[
\frac{M_{\text{max}} \cdot y_T}{I_{\text{gross}}} = \frac{GA}{NA} = \frac{227.813 \times 10^6 \times 20.8 \times 1.4}{304.89 \times 10^6} = 217.58 \gg 16.5
\]

Let us adopt area of plate $1800 \text{ mm}^2$ and staggering of rivets hence

\[
B \times t = 1800 \text{ mm}^2
\]

\[
\text{adopt } t = 10 \text{ mm}
\]

\[
B = 180 \text{ mm}
\]

\[
\frac{Ga}{Na} = \frac{1800}{1800 - 21.5 \times 10} = 1.136
\]

\[
I_{\text{gross}} = 205 \times 10^6 + 2 \left[ \frac{180 \times 10^3}{12} + 1800 \times 20.5^2 \right] = 356.32 \times 10^6 \text{ mm}^4
\]
Check for bending: \[ \frac{My}{I} \times \frac{5A}{NA} \]

\[ = \frac{22.7 \times 81.3 \times 10^6 \times 210 \times 1136}{3.56 \times 32 \times 10^6} \]

\[ = 152.52 \text{ N/mm}^2 < 165 \text{ N/mm}^2 \]

Check for shear:

Area = 720 \times 8.9 = 3788 \text{ mm}^2

Av. shear stress = \[ \frac{V_{\text{max}}}{A} \]

\[ = \frac{135 \times 10^6}{3728} = 36.12 \frac{\text{N}}{\text{mm}^2} < 100 \]

Check for deflection:

\[ \Delta_{\text{max}} = \frac{5 \times 40 \times 6.75 \times 10^4}{384 \times E \times I} \]

\[ = \frac{5 \times 40 \times 675000}{384 \times 2 \times 10^5 \times 356.32 \times 10^6} \]

\[ = 15.17 \text{ mm} \]

\[ \frac{\text{Span}}{3.25} = \frac{6.75}{3.25} = 2.077 \]

\[ 15.77 < 20.77 \]

Note: This is from the compressive point of view.
Check for secondary criteria

Assuming $g = 80\text{mm}$

$h_2 = 32.8\text{mm}$

Check for local buckling of flange plate

$b = \frac{180 - 80}{2} = 50\text{mm}$

$\frac{b}{t} = \frac{50}{10} = 5 < 16$

Check for web crippling

The beam is carrying only one hinged point loading will only be at support hence web crippling will be checked at support.

\[
\frac{13.5 \times 10^3}{8.9 \left( \frac{2.10 + \sqrt{3}}{2} \right) (31.8 + 10)} 
\leq \frac{75}{f_d}
\]

53.39 \leq 187.5

Safe in web crippling
Check for web buckling

\[ f = \frac{dw \sqrt{3}}{tw} = \frac{(D - 2h_2) \sqrt{3}}{tw} \]
\[ = \frac{(400 - 2 \times 32.8) \sqrt{3}}{8.9} = 65.07 \]

\[ \sigma_{\text{al}} = 110.207 \text{ N/m}^2 \]

Area of resisting section
\[ = (210 + 210) \times 8.9 \]
\[ = 3738 \text{ mm}^2 \]

Carrying capacity
\[ = 3738 \times 110.207 = 4119.95 \text{ kN} \]
\[ > 135 \text{ kN} \quad \text{Safe} \]

Rivet Design

\[ \frac{V_A}{A} \leq f_V \]

because no. of rivet forPitch length = 1

180 x 10 = 1800 mm²

\[ f_V = 201 \text{ kN/m} \]

\[ f_{\text{max}} = 356.3 \times 10^6 \]

\[ V = 175 \times 10^3 \text{ N} \]

\[ P_{\text{min}} \text{ dia} = 20 \text{ mm} \]

\[ k_v = 3.63 \text{ kN} \]
\[ P \leq 259.68 \, \text{mm} \]

Note:

16t \times 12t \times 200 \, \text{mm} \text{ is a recommendation for closed}

loading. In this case, limit would be 32t or

300 \, \text{mm}. Hence, max. pitch will be

\[ \frac{160}{\text{min}} \text{ or } 32t, 300 \, \text{mm} \]

Max. pitch = 300 \, \text{mm}

Adopt \[ P = 250 \, \text{mm} \]

The pitch can be increased towards the midspan. Here,

even we take it constant through out.

Permissible stresses

\[ \bar{\sigma}_{te} = \bar{\sigma}_{bt} = 0.67 \, \text{fy} \]

\[ \bar{\sigma}_{shea} = 0.4 \, \text{fy} \]

\[ \bar{\sigma}_{bt} = 0.75 \, \text{fy} \]

All these permissible values are increased by \( 33 \frac{1}{3} \% \) if

effect of wind or \( E_0 \sigma_0 \) is taken into account.
IS 120 @ 79.4 kgf/m

1000 kgf = 10 kN

\[ f = 10 \]

\[ \frac{W}{10} \]

\[ \frac{W}{10} \]

for the restrained beam like this max. b.m. will occur at the support

\[ \frac{P_1 + \frac{Wt^2}{12}}{8} \]

\[ \frac{P_1 + \frac{Wt^2}{12}}{12} \]

\[ W \]

\[ \frac{W}{8} \]

\[ \frac{W}{10} \]

\[ \frac{P_1 + \frac{Wt^2}{12}}{8} = 14.235 \text{ kNm} \]

\[ \frac{W}{8} = \text{ W kN}\]

Tension (tensile)

\[ \sigma = \frac{14.235 \times 10^6 \gamma}{\bar{I}_{yx}} + \frac{1 \times 10^6 \times x_k}{\bar{I}_{yy}} \]

at N.A. bending stress in kN/m

\[ \gamma \]
\[
\frac{19.235 \times 10^6}{(\frac{I_{xy}}{y})} + \frac{\omega x 10^6}{(\frac{I_{yy}}{x})} = 165 \text{ MPa}
\]

\[
\frac{19.235 \times 10^6}{15581 \times 10^2} + \frac{\omega x 10^6}{170.7 \times 10^2} \leq 165
\]

\[
\omega \leq 26.6 \text{ kN}
\]

Location of N.A. shows B. stress will be maxm either at A or B.

Instead of finding out location of N.A. by above approach, we could have located it under the concept that N.A. lies b/d minor axis and the direction of resultant moment for safety, \(\sigma_a \leq \sigma_{perm}\).
Grillage Foundation

Top truss beam

Bottom box

I 5 4B

I 5 6B

Encased in concrete

Permissible shear

If the grillage foundation is encased in concrete, the permissible shear are increased by \(33\frac{1}{3}\%\).

If effect of wind and ED is also taken into account, permissible shears are increased by 50%.

Note: Bending stress is normally is \(0.67 \text{ kN/m}^2\).

For all sections, \(f_{c,ed} = \frac{0.33 \times E}{3} \text{ kN/m}^2\).

For bending stresses in grillage foundation when effect of wind or ED is also taken into account, \(f_{c,ed} = 1.5 \times 0.67 \text{ kN/m}^2\).
Separation must keep the beam properly spaced such that the distance between edges of adjacent beam is not less than 75 mm.

\[ \frac{f}{L \cdot B} \leq \text{bearing capacity of soil} \]

Choose \( L \) & \( B \) on this criteria

**Top tier Beams**

\[ M_{\text{max}} = \frac{1}{2} \cdot \frac{f}{L} \left( \frac{L}{2} \right) \times \frac{1}{2} \]

\[ M_{\text{max}} = \frac{f}{8} \left( L - \alpha \right) \]

The above max \( f \) & \( Bm \) are for all top tier beams taken together. Hence to design top tier beam no. 1 "}


beam needs to be selected.

To start with area of base plate can be calculated as follows

\[ P = \text{Area} = a \times b \]

Once the size of base plate is known no of top tier beam can be selected, no of top tier beam should be such that it could be accommodated within width b.

Hence Max. B.M. in one top tier beam:

\[ \frac{P(l-a)}{8n} \]

Where n = no of top tier beam.

Max. S.F. in one top tier beam:

\[ \frac{P(l-a)}{2l \times n} \]

\( P \) is the load in which effect of dead wt already been taken into account.

\[ Z_{req} = \left\{ \frac{P(l-a)}{8 \times n} \right\} \]

\[ 165 \times 1.33 \]

Choose seen and check for shear.

Bottom Tier beam:

\[ \frac{P}{b} \]

Mwb. for all bottom tier beam taken together:

\[ \frac{P(b-0.5)}{8} \]
\( V_{nm} \) for all bottom tier beam taken together

\[
V_{nm} = \frac{P(8 - h)}{2h}
\]

Note: If there is a chance of web crippling it will be there in top tier beam only hence top tier beam can be checked for web crippling also.

Load on a column is 2500 KN, the safe bearing capacity of soil is 250 KN/m² and permissible bearing stress in conc. is 4000 KN/m². Design grillage foundation.

Set: Load on base plate = 1.1 x 2500 = 2750 KN

Cheecking square base plate

\[
0 \times a = \frac{1.1P}{c} = \frac{2750}{4000} = 0.6875
\]

\[
a = 0.829
\]

\[
a = 850 \text{ mm}
\]

\[
\frac{1.1P}{25} = L^2
\]

\[
L = \frac{1.1 \times 25}{6.6} = 3.316 \text{ m}
\]

\[
\text{ adept } = 3.4 \text{ m}
\]

\[
M_{max} = \frac{1.1P}{8} (L - a) = -876.56 \text{ KNm}
\]

\[
V_{max} = \frac{1.1P}{2L} = 1031.25 \text{ KN}
\]

\[
2 \text{ seq for all beams taken together}
\]

\[
Z_{seq} = \frac{876.56 \times 10^6}{165 \times 1.33} = 4000 \text{ cm}^3
\]
Let us choose 3 no. of top tier beams

\[ 2 \times \text{eq. for } 1 = \frac{4000}{3} = 1333.33 \]  

2. Let us choose ISMB 450

\[ b = 150 \]  

So 3 no. of ISMB 450 can be accommodated within the base plate

\[ \frac{1.1 \times 0.75 \times (1 - a)}{2 \times 3} = 343.75 \text{ KN} \]

Av. shear stress \( \frac{313.3}{950 \times 1.4} \)

\( = 81.26 < 125 \)kip

\( 35 \text{ ft} \)

\[ \frac{1.1 \times 2500}{3} = 916.7 \text{ KN} \]

\[ \frac{916.7 	imes 10^3}{(0.25 + 3) h_2} \leq 1.33 \times 0.75 \text{ kip} \]

\[ 100.26 \leq 247.175 \text{ kip} \]

\( 100 \)
Max. spacing b/t bottom tie beam is generally 200 to 250 mm. 

No. of bottom tie beam

Max. bottom tie for all beams taken together

\[
Z_{req.} = \frac{P(B-H)}{8} \approx 876.56 \text{ KNm}
\]

\[
\frac{876.56 \times 1.33}{133 \times 16.5} = 4000 \text{ cm}^3
\]

Space available = 3.9 m

No. of bottom tie beams are generally 10 - 20

Let us choose 15 no.

\[
Z_{req. \text{ one}} = \frac{4000}{15} = 266.7 \text{ cm}^3
\]

Choose ISMB 225

\[
Z_{\text{provided}} = 305.9 \text{ cm}^3
\]

14 x 75 + 15 x 110 < 3400

2710 < 3400 hence can be accommodated

Check for shear:

Shear in one bottom shear beam

\[
\frac{1031.25}{15} = 68.75 \text{ KN}
\]

Av. shear stress

\[
\frac{68.75}{6.5 \times (22.5) \times 100} = 47 \text{ N/mm}^2
\]

\[
\leq 1.33 \times 100
\]

By
Note: If the size of column is given to us, thickness can be calculated.
In conventional design, method structure is designed for strength of steel till it yields at a point in the shear.

However, if yielding occurs at one point in a section, it does not mean collapse of the member. Due to plastic deformation and strain hardening of material, the particles which were less stressed will be brought into action so that the structure is actually able to resist greater load.

In the modern design, strength of steel beyond the point of first yield is utilized and this method of design is called plastic method of design.

Simply stress-strain curve.

Strain hardening range has been omitted in this case which, in fact, will add to the margin of safety.

Plastic design can be applied only to redundant structures.

In the case of simple members, load causing first yield is most critical because at that load only large unacceptable deformation occurs.
str. deformation even at the time of collapse is not much
hence plastic can be applied only to redundant str.

Assumptions for plastic analysis

1. Material should possess ductility so that it can be deformed
   in the plastic stage.
2. Strain distribution is linear.
3. Relation b/w tensile stress and tensile strain & comp. strain is
   some.
4. Joints should be sufficiently strong to transfer the moment.
   All joints should be rigid joint.

Plastic Bending of Beams

\[ C = \frac{1}{2} f_t (D-x) b \]
\[ T = \frac{1}{2} f_t x b \]
\[ f_c (D-x) y = f_t x b \]
\[ (D-x)^2 = x^2 \]
\[ b(D-x) \cdot (D-x) = \frac{b x \cdot x}{2} \]

Moment of comp. area about N.A = Moment of ten. area about N.A

This is the location of C.G.
N.A. will remain at centroidal axis if section is symmetrical about the axis of bending. (in fig 8.4.8)

In fig (8)

\[ C = T \]
\[ f_y \Delta c = f_y \Delta t \]
\[ \Delta c = \Delta t \frac{A}{2} \]
N.A. = equal area axis

\[ C_B = T \cdot B = M_p \]

\( M_p \) is full plastic moment capacity of section.

- \( M_p \) is plastic moment capacity which depends on section only.

A fully plastic section can not resist any further moment and if loading is applied beyond \( M_p \), unrestrained rotation will take place at the fully yielded section thus we assume as if a plastic hinge has formed. Plastic hinge can be understood as rusted hinge in which rotation does not take place up to certain load and beyond that load a large deformation in the form of rotation will take place.
Formation of plastic hinge at one location does not mean the collapse of the structure. For collapse sufficient no. of plastic hinge must develop to make the structure unstable.

\[ m_x \]
\[ m_y \]
\[ \text{Fixed end stubs behaving in hinge} \]

For complete collapse of structure the margin available is \((W_s - W_h)\) beyond the point of first yield \((W_y - W_x)\).

For complete yielding of the sec\(n\) margin available beyond the point of first yield is \((W_y - W_h)\).

Definition of Plastic hinge—A plastic hinge can be defined as a yielded zone due to flexure in a structure in which infinite rotation can take place at a constant resisting moment \((m_p)\) of the sec\(n\).

Length of plastic hinge:

\[ \text{Yielded zone} \]

\[ P \]
\[ P \]
\[ m_x \]
\[ m_y \]
Some important points about plastic hinge:

1. A section is said to develop a plastic hinge when due to flexural stress at every point of the section is equal to yield stress.

2. Plastic hinge develops first at section subjected to greatest curvature.

3. Due to formation of plastic hinge one after the other, redistributing of moments take place. Sufficient no. of plastic hinges have to be developed to render the structure to unstable or collapse state.

4. No. of plastic hinge required for complete collapse of the structure is \((\delta + 1)\).

   Where \(\delta\) is the degree of redundancy.

\[ \begin{align*}
\delta_1 = 2 & \quad \text{for vertical loading} \\
\delta_2 = 2 & \quad \text{for partial collapse of } \delta_2 \text{.}
\end{align*} \]

No of plastic hinge required for complete collapse of \(\delta_2 = 3\) + 1 = 4

No of plastic hinge for complete collapse = 2 + 1 = 3

Length of plastic hinge depends on loading & shape of section.
6) for purpose of analysis plastic hinge will be assumed as a point about which plastic rotation takes place.

7) m.o.e. of normal hinge is zero & m.o.e. of p.h. is $M_p$

8) Plastic hinge is expected to form at
   a) fixed ends
   b) at the location of point load.
   c) at the point of sudden change in geometry.
   d) at points of zero shear in a span subjected to u.d.l. or u.v.l.

9) when two sections join at a point plastic hinge forms in a section of smaller $M_p$

If section is loaded up to a moment of $M_y$ and then unloaded complete recovery will take place. However, if moment applied on the section is $> M_y$, then unloading takes place recovery will not be complete.
Shape factor:

$$\text{Shape factor} = \frac{M_p}{M_y}$$

It shows the reserve of strength of a section beyond the point of first yield.

In elastic range:

$$M_y = f_y \cdot Z$$

In plastic range:

$$M_p = \frac{C(\overline{y_1} + \overline{y_2})}{f_y} = \frac{C}{f_y} \cdot \frac{A}{2} (\overline{y_1} + \overline{y_2})$$

$$M_p = \frac{C}{f_y} \cdot S \quad \text{where} \quad S = \text{plastic modulus} \quad = \frac{A}{2} (\overline{y_1} + \overline{y_2})$$
Shape factor:
\[
\frac{A}{\bar{y}} \left( \overline{y_1} + \overline{y_2} \right) \frac{S}{2} = \frac{S}{2}
\]

Plastic modulus:
\[
\frac{A}{2} \left( \overline{y_1} + \overline{y_2} \right) \frac{b d}{2} = \frac{b d^2}{2}
\]

Circular section:
\[
\frac{A}{2} \left( \overline{y_1} + \overline{y_2} \right) \frac{d}{2} = \frac{\pi d^2}{8} \left( \frac{4 \cdot (0 \cdot \delta)}{3 \pi} \right)
\]

Plastic modulus for circular section:
\[
\frac{1}{6} \frac{1}{8 \times \frac{4}{3}} = \frac{1}{16} \approx 0.0625
\]

Shape factor of rectangular section:
\[
S_F = \frac{A}{2} \left( \overline{y_1} + \overline{y_2} \right) \frac{b d}{2} = \frac{b d^2}{6}
\]

Circular section:
\[
S_F = \frac{A}{2} \left( \overline{y_1} + \overline{y_2} \right) \frac{d}{2} = \frac{\pi d^2}{8} \left( \frac{4 \cdot (0 \cdot \delta)}{3 \pi} \right)
\]

Plastic modulus for circular section:
\[
\frac{1}{6} \frac{1}{8 \times \frac{4}{3}} = \frac{1}{16} \approx 0.0625
\]

Circular section:
\[
S_F = \frac{A}{2} \left( \overline{y_1} + \overline{y_2} \right) \frac{d}{2} = \frac{\pi d^2}{8} \left( \frac{4 \cdot (0 \cdot \delta)}{3 \pi} \right)
\]

Plastic modulus for circular section:
\[
\frac{1}{6} \frac{1}{8 \times \frac{4}{3}} = \frac{1}{16} \approx 0.0625
\]
\[
S = \frac{A}{2} \frac{y_1 + y_2}{z} = \frac{A}{2} \left( \frac{a^4}{\sqrt{2}} \frac{1}{3} x z \right) \frac{1}{12} a = \frac{a^3}{3 \sqrt{2}} \\
S = \frac{\frac{1}{2} \left( y_1 + y_2 \right)}{z} = \frac{S}{z} \\
S = \text{plastic modulus} = \frac{d^3}{6} - \left( \frac{d - 2t}{d} \right)^3 = \frac{d^3}{6} \left[ 1 - \left( \frac{d - 2t}{d} \right)^2 \right] \\
= \frac{d^3}{6} \left[ 1 - \left( 1 - \frac{2t}{d} \right)^2 \right] = \frac{d^3}{6} \left[ 6 \cdot \frac{1}{d} \right] = d^2 t \\
Z = \frac{1}{\frac{d}{2z}} = \frac{\pi d^4}{64} - \frac{T (d - 2t)^4}{64} \frac{d}{z}
\[
\frac{\eta d^2 t}{32} \left[ 1 - \left(1 - \frac{2t}{d}\right)^{\frac{1}{3}} \right]
\]
\[
\frac{\eta d^2 t}{32} \left[ 1 - (1 - \frac{8t}{d} + \ldots) \right]
\]
\[
\frac{\eta d^2 t}{32} \times \frac{8t}{d} = \frac{\eta d^2 t}{4}
\]

S.F. = \frac{d^2 t}{\eta d^2 t} \times 4 = 1.273

\[
\sum_{n=1}^{T} \frac{1}{n^2} \frac{1}{a^2} \frac{1}{\gamma} = \frac{5}{2}
\]

\[
2 = \frac{\gamma}{\gamma} = \frac{a \cdot (\frac{a}{\gamma})^2}{2 \cdot 4} \frac{a^3 \cdot \gamma^2}{4 \times 2 \cdot 4} \times 3 \times \frac{1}{3}
\]

\[
a \cdot \text{Area} = \frac{1}{2} \left\{ \text{Area} \right\} = \frac{\gamma}{2} = \frac{a^2 \frac{\sqrt{3}}{4}}{2}
\]

\[
\text{Area of equilateral triangle} = \frac{a^2 \frac{\sqrt{3}}{4}}{4}
\]

\[
\frac{x^2 \frac{\sqrt{3}}{4}}{4} \times \frac{a^2 \frac{\sqrt{3}}{4}}{4} \times \frac{1}{2}
\]

\[
\sum_{k=1}^{\infty} \left( \frac{2k+1}{\sqrt{n} + 1} \right) \frac{\sqrt{n}}{2} \left( \frac{\sqrt{n} - 1}{2} \right) = \frac{19x}{18} \times \frac{268}{2}
\]
Length of plastic hinge

\[
\frac{m_p}{m_y} = \frac{\frac{f}{2}}{\left(1 - \epsilon_p\right)^2} = 1.5
\]

\[
\epsilon_p = \frac{f}{3}
\]
\[
\frac{M_p}{M_y} = \frac{wL^2}{8} = \frac{wL^2 - wLp^2}{2} - \frac{L^2}{2}
\]

\[
l^2 = 1.5L^2 - 1.5LP^2
\]

\[
lp^2 = \frac{L^2}{3}
\]

\[
lp = \frac{L}{\sqrt{3}}
\]

Load factor and factor of safety

Load factor = \frac{\text{collapse load}}{\text{working load}}

Note:

Plastic Design

\[
(5 \times h_s \times \text{load factor})
\]

\[
(5 \times h_s) \times L^2 \leq M_p
\]

Elastic Design

\[
\frac{wL^2 + y}{8} \leq f_y
\]

\[
\frac{wL^2}{8} \leq f_y
\]

\[
f_y
\]

\[
f_y
\]

\[
f_y
\]

\[
f_y
\]

\[
f_y
\]
Load factor = \frac{\text{collapse load}}{\text{working load}} = \frac{W_k}{W_w}

= \frac{M_p}{\alpha_{rej} - 2} \frac{f_y, s}{(\frac{f_y}{f_{o.s}})^2} \frac{R_0 S \times (\frac{3}{2})}{2}

Load factor = \text{(factor of safety)} \times \text{shape factor}

\text{Moment curvature relationship (Rectangular section)}

So long as moment is less than \( M_y \)

Elastic condition prevails

\( \frac{M}{I} = \frac{f}{y} = \frac{E}{k} \)

\( M = EST = ES \frac{k}{L} \), curvature

\( \frac{M}{M_y} = \frac{I_k}{k_y} \)

\( \frac{f}{f_y} \)

As the section starts yielding, moment-curvature relationship becomes non-linear.

Even in the yielded zone, the strain variation is assumed...
to be linear.

\[ \varepsilon = \frac{y}{y_0} = \varepsilon_k \]

\[ k = \frac{\varepsilon}{y} \]

\[ \text{Strain at } A = \frac{f_y}{E} = K \varepsilon \]

\[ \text{Curvature} \left( \frac{y}{y_0} \right) = \left\{ \frac{f_y}{E} \frac{h}{2} \right\} \]

\[ \frac{k}{Ky} = \frac{h}{2c} \]

\[ M = c_1 \alpha_1 + c_2 \alpha_2 = T_1 \alpha_1 + T_2 \alpha_2 \]

\[ c_1 = \frac{f_y}{E} \left( \frac{h}{2} - c \right) b \]

\[ a_1 = h - \left( \frac{h}{2} - c \right) = \left( \frac{h}{2} + c \right) \]

\[ c_2 = \frac{1}{2} f_y \cdot c \cdot b \]

\[ \alpha_2 = \frac{4c}{3} \]
Question: Design a simply supported beam of span 6m to support a point load of 100 kN acting at its mid-span.

\( f_y = 250 \text{ MPa} \) and load factor = 1.75
collapse load = 100 x 1.75 = 175 kN

Beam will collapse when plastic hinge forms at midspan below the point load.

b.m. at A = \( M_p = \frac{175 \times 8}{4} = 262.5 \text{ kNm} \)

\[ M_p = f_y \cdot S \]

\[ S = \frac{262.5 \times 10^6}{250} = 10.5 \times 10^3 \text{ mm}^3 \]

\[ \frac{S}{Z} = \text{shape factor} \]

\[ Z = \frac{S}{sf} \]

By choosing shape factor, i.e., by choosing the secant shape \( Z \) can be calculated.

Note

shape factor for I sec = 1.15

\[ Z_{e q} = \frac{10.5 \times 10^3}{1.15} = 9.13 \times 10^3 \text{ cm}^3 \]

Choose

I5MB - 900

\[ Z = 1022.9 \text{ cm}^3 \]
Important theorems in plastic analysis
In the plastic analysis following conditions must be satisfied.

1. Equilibrium condition

\[ \Sigma F = 0 \]
\[ \Sigma M = 0 \]

In all types of analysis equilibrium eq. is always satisfied.

2. Mechanism condition

At collapse sufficient no of plastic hinge must be develop so as to transform a part or whole of the structure into a mechanism leading to collapse.

For complete collapse of the structure no of plastic hinge required to be formed = \( n \times (n+1) \)

\[ n = \text{Degree of redundancy of structure} \]

3. Yield condition

At collapse B.M. at any section must not exceed the fully plastic moment capacity of the section

If all the above 3 cond. are satisfied, a unique lowest value of collapse load will be achieved.

Based on the above 3 cond. we get the following theorems based on which plastic analysis is performed.

\( \text{a) Upper Bound theorem, or kinematic theorem} \)
\( \text{b) Lower Bound theorem or static theorem} \)
Upper Bound Theorem. This theorem satisfies equilibrium and mechanism condition. Load determined by assuming a mechanism will always be greater than equal to the collapse load \( P \geq P_c \). \( P_c \) is collapse load.

This theorem can also be stated as,

Of the various possible mechanism, the correct mechanism is the one for which the collapse load is min.

Lower Bound Theorem.

Load determined on the basis of any collapsed B.M.O., in which B.M. at any w.t. is less than plastic moment, will be less than or equal to actual collapse load i.e. \( P \leq P_c \).

This theorem satisfies equilibrium and yield cond.

Methods of Analysis:

a) Static Method.

b) Kinematic Method.

Static Method:

Steps:

1) Select the redundant force. Moment will be taken as redundant.

\[ D_3 = 1 \]
3) Draw the B.M.D. and redundant B.M.D.

3. A revolved B.M.D is drawn in such a way that a mechanism is formed.

4) Collapse load is found out by working out the equation

5) It is checked that B.M. at every such is less than M_r
Kinematic Method.

Locate the possible places of plastic hinges and ascertain the various possible mechanisms.

The collapse load is found out by applying the principal of virtual work.

BMD of collapse mechanism is drawn and it is checked that θ is at any sec is not more than θₚₙ. 189

No. of plastic hinge required = 2

\[ M_p \theta - M_p \theta - M_p \theta \cdot \frac{w_c \theta \lambda}{2} = 0 \]

\[ -3M_p \theta = \frac{w_c \theta \lambda}{2} \]

\[ w_c = \frac{6M_p}{\lambda} \]
Kinematic Method.

Locate the possible places of plastic hinges and ascertain the various possible mechanisms.

Collapse load is found out by applying the principle of virtual work.

BMD of collapse mechanism is drawn and it is checked that BMA at any seen is not more than \( M_p \).

No. of plastic hinges required = 2

\[
\begin{align*}
\sum M &= 0 - M_p \theta - M_p \theta - M_p \theta - \psi, \quad \sum \psi &= 0 \\
&= 3M_p \theta - W_c \frac{0 \cdot l}{2} \\
W_c &= \frac{6M_p}{l}
\end{align*}
\]
0) Find \( M_p \) if \( c \) is the collapse load

1) Find the position of load for collapse load to be min.

\[ R = 1 \quad (\text{for vertical loading}) \]

No of Hinge required for complete collapse \( = n + 1 = 2 \)

\[ M_p + \frac{M_p - b}{a+b} = \frac{cab}{(a+b)} \]

\[ M_p = \frac{cab}{(a+2b)} \]

---

**Kinematic method**

\[ -M_p \theta - M_p \theta - M_p \phi + c - b \phi = 0 \]

\[ a \theta = b \phi \]

\[ -2 M_p \theta - M_p \frac{a \theta}{b} + c a \theta = 0 \]

\[ M_p = \frac{cab}{a+2b} \]
\( \frac{c}{a+2b} = \frac{m_p}{ab} \)

\[
= \frac{m_p (l + b)}{(l - b) b}
\]

for \( c \) to be min

\[
\frac{dc}{db} = 0
\]

\[
\frac{m_p (l - b) b - m_p (l + b) (l - 2b)}{(l - b) ^2 b} = 0
\]

\[
(l - b) b - (l + b) (l - 2b) = 0
\]

\[
2b - b^2 - (l^2 - 2b^2) = 0
\]

\[
2b - b^2 - l^2 + b^2 = 0
\]

\[
b^2 + 2b - l^2 = 0
\]

\[
b = \frac{-2 \pm \sqrt{4 + 8l^2}}{2}
\]

\[
b = l \pm \sqrt{l^2 - 1}
\]

\[
b = 0.414 + l
\]

That in case of hinged cantilever with point load collapse load will be min when load is at 0.414 \( \& \) from the hinged end.
Method 1

The plastic hinge will form at support and second plastic hinge will form somewhere in span, the location of plastic hinge will be at the point of zero shear because the beam is loaded with U.O.C. L.

\[ M_p \]

\[ SF = 0 \text{ at } x \text{ distance from hinged end} \]

\[ V - Cx = 0 \]

\[ Vx - \frac{Cx^2}{2} = M_p \]

\[ Cx^2 - \frac{Cx^2}{2} = M_p \]

\[ \frac{Cx^2}{2} = M_p \]

At support A

\[ M_1 - \frac{Cx^2}{2} = -M_p \]

\[ Cx - \frac{C_1}{x} = -Cx^2 \]

\[ 2M_p = C \left( \frac{L-x}{2} \right)^2 = 0 \]

\[ Cx^2 - C \left( \frac{L-x}{2} \right)^2 = 0 \]

\[ 2x^2 - L^2 - x^2 + 2Lx = 0 \]

\[ x^2 + 2Lx - L^2 = 0 \]

\[ x = 0, 4.14L \]
\[ x^3 + 2.8x - 1.2 = 0, \]
\[ x = 0.414 \text{ ft} \]

By Kinematic Method

\[ -M_p \theta - M_p \phi - M_p \dot{\phi} + C \frac{1}{2} l \cdot (l-x) \theta = 0 \quad (1) \]
\[ \ddot{\phi} - \frac{(l-x)}{x} \theta = 0 \quad (11) \]
\[-2M_p \theta - M_p (l-x) \theta + C \frac{\ell (l-x) x}{2} = 0 \]
\[-M_p (2 + \frac{l-x}{x}) + \frac{Cy (l-x) x}{2} = 0 \]
\[-M_p \frac{(l+x)}{x} + \frac{Cy (l-x) x}{2} = 0 \]

\[ \frac{Cy (l-x) x}{2} = \frac{\ell (l-x) x}{2} \]

for \( M_p \) to be max. \( \frac{dM_p}{dx} = 0 \)

\[ \begin{aligned}
\frac{Cy (l-x) x}{2} & - \frac{2(l-x)(l-x)}{2(l+x)(l-x)} \times \frac{Cy (l-x) x}{2} = 0 \\
\{ 2(l+x)^2 \} & - \{ 2(l-x)^2 \} = 0 \\
(l+x)(l-x) & - (l-x) x = 0 \\
(l+x)x - (l-x)x & = 0 \\
l^2 - 2lx + l^2 - 2x^2 - l^2 + x^2 & = 0 \\
x^2 - 2lx + l^2 & = 0 \quad \Rightarrow x = 0.414 \text{ ft} \]

Note: \( M = f(x) \)

We calculate the max value of \( M \)

& maximum value of \( M \) is \( M_p \)

So we have \( \frac{dM_p}{dx} = 0 \)
\[ M_p = \frac{c x^2}{2} \]

\[ = c \left( \frac{4.14}{2} \right)^2 = \frac{(0.085)^2}{2} = \frac{(3 - 2.65)}{2} \cdot c \cdot l^2 = 0.6863 \cdot (c \cdot l^2) \]

Find value of \( a \) for simultaneous collapse of \( AB + AD \)

\text{collapse RMD}

For simultaneous collapse of \( AB + BD \) plastic hinge should form at \( B \), at \( A \) & in b/w \( A + B \)

\( \text{Note:} \)

Plastic hinge forms at \( \cdot 4.14 \cdot l \) when moment at the hinged end = 0. But if there is moment at the hinged end plastic hinge will not form at \( 0.414 \cdot l \)
\[
\frac{ca^2}{2} = M_t \\
\frac{cl^2}{8} = 2M_t \\
\frac{cl^2}{8} = 2\frac{ca^2}{2} \\
a^2 = \frac{l^2}{8} \\
a = \frac{l}{2\sqrt{2}} = 0.353l
\]

Note:

If \( a < 0.353l \),
ARB will collapse.

If \( a = 0.353l \),
\( AB \) and \( BD \) will collapse.

If \( a > 0.353l \),
\( BD \) will collapse.
Plastic moment = $M_p$

find collapse load = $w$

$R = 1$

No of P.M. for complete collapse = 1 + 1 = 2

one will fail at support and other at $x$ dis from A

$2M_p - \frac{w(l-x)^2}{2} = 0$

$\Sigma M_B = 0$

from (I) and (II)

$\frac{w(l-x)^2}{2} = \frac{wx^2}{2} - \frac{wx^2}{18}$

$(l-x)^2 = 2x^2 - \frac{2l^2}{9}$

$9(l-x)^2 = 18x^2 - 2l^2$

$9[l^2 + x^2 - 2lx] = 18x^2 - 2l^2$

$11l^2 - 9x^2 + 18lx = 0$

$9x^2 + 18lx = 11l^2 = 0$

$x = 0.4907l$

by putting $M_p = \frac{wx(l-x)^2}{4}$

collapse load $w = \frac{4M_p}{(l-x)^2} = 15.421 \frac{M_p}{l^2}$
\[(g-x) \phi = \phi \frac{d}{dx} \phi \]

\[-m_p \frac{\partial}{\partial x} ( \phi ) + \frac{1}{2} \omega \frac{\partial}{\partial x} \phi - \frac{1}{2} \omega \frac{1}{3} \frac{\partial^2}{\partial x^2} \phi = 0 \]

\[-m_p \frac{\partial}{\partial x} ( \phi ) + \frac{\omega}{2} \frac{1}{2} \frac{\partial^2}{\partial x^2} (x-x) \phi - \frac{\omega-1}{18} \frac{(x-x) \phi}{x} = 0 \]

\[-m_p \frac{\partial}{\partial x} \left[ \frac{f+x}{x} \right] + \frac{\omega-1}{2} (x-x) \phi - \frac{\omega-1}{18} (x-x) \phi = 0 \]

\[m_p = \frac{\omega (x-x) \left[ 1 - \frac{x}{q} \right]}{\left( \frac{f+x}{x} \right)} \]

\[= \frac{\omega (x-x) (q x - f) \left( \frac{f+x}{x} \right)}{18 (x+f)x} \]

\[\frac{k}{x} \frac{(f-x)(q x - f)}{(x+x)} \]

\[\frac{\partial m_p}{\partial x} = 0 \]

\[\frac{k}{x+f} \left( q x^2 - 10 f x - 1^2 \right) \]

\[(f+x) (-18 f x + 10 f) - (-q x^2 + 10 f x - 1^2) = 0 \]

\[-18 f x + 10 f^2 - 18 x^2 + 10 f x + q x^2 - 1 y x + x^2 = 0 \]

\[-q x^2 - 18 f x + 11 x^2 = 0 \]
Beam of uniform cross-section

$M_P$ is constant

$R = 2$

No. of P.M. = 3

find collapse load $w$

3 hinges will form 2 at the support 4

will form b/w A B b/c with S.F. will be zero only in A B

\[
\begin{align*}
\text{sum} & = 0 \text{ of left FBD} \\
2M_P - \frac{wx^2}{2} & = 0 \\
M_P & = \frac{wx^2}{4} \\
4x^2 & = \frac{w}{k} \left[ 3l^2 - 2lx - 6lx + 4x^2 \right] \\
8lx & = 3l^2 \\
x & = \left( \frac{3l^2}{8} \right) \\
\text{collapse load} \ w & = \frac{4M_P}{x^2} = \frac{4M_P \times 64}{9l^2} = \frac{256 M_P}{9l^2} = 28.9 + M_P
\end{align*}
\]
By kinematic method.

\[
\begin{align*}
-m_p \dot{\theta} - m_p \dot{\phi} - m_p \phi - m_p \dot{\phi} + \frac{1}{2} \omega \left( \frac{x}{l} - x \right) \left( \frac{1}{l} \phi \right) + \left( \frac{1}{l} \phi \right) \left( \frac{1}{l} \phi \right) &= \frac{1}{2} \omega x \phi \\
-2 m_p \left( \frac{x}{l} \right) \left( \frac{l-x}{l} \right) - 2 m_p \phi + \frac{1}{2} \omega \left( \frac{x^2}{l^2} - x \right) \left( \frac{1}{l} \phi \right) + \frac{1}{2} \omega \phi &= 0 \\
-2 m_p \left( \frac{x^2}{l^2} - x \right) \left( \frac{l-x}{l} \right) - 2 m_p \phi + \frac{1}{2} \omega \left( \frac{x^2}{l^2} - x \right) \left( \frac{1}{l} \phi \right) + \frac{1}{2} \omega \phi &= 0 \\
+2 m_p \left( \frac{x^2}{l^2} - x \right) \left( \frac{l-x}{l} \right) - 2 m_p \phi + \frac{1}{2} \omega \left( \frac{x^2}{l^2} - x \right) \left( \frac{1}{l} \phi \right) + \frac{1}{2} \omega \phi &= 0 \\
4 m_p \left( \frac{l}{l-x} \right) & = \frac{\omega}{8} \left( l^2 + 4lx \right) \\
\omega & = \frac{32 m_p}{l} \left( \frac{l}{l-x} \right) \left( \frac{l^2 + 4lx}{l^3 + 4l^2x - 2lx^2 - 8lx^2} \right) \\
\frac{d\omega}{dn} &= 0 \\
\left( \frac{dx^2}{l} - 2 \frac{x^2}{l} - 1lx \right) &= 0 \\
2 \frac{x^2}{l} - 1lx &= 0 \\
\frac{x^2}{l} &= \frac{1}{8} \\
\phi &= \frac{4}{8} - \frac{1}{8} = \frac{a-1}{8} \\
A_{-2} & = \left( \frac{2}{l} \right) \\
A_{-1} & = \left( \frac{1}{l} \right)
\end{align*}
\]
Note: In case of fixed beam with constant cross-section, the location of plastic hinge in the span will be at the location of max. moment in free B.M.
Find collapse load $w$

Degree of redundancy = 2

No. of p.n. eqn. = 3

Case 1: mode of failure

When two members join at a point, plastic hinge form in the member of smaller $M_p$.

$$5M_p + \frac{1}{2} \cdot 8 \cdot \frac{8}{2} \cdot w = 0$$

$$w = \frac{40M_p}{g^2}$$

Case 2:

$$\begin{align*}
2M_p &= \frac{w(x)}{L} \\
M_p &= \frac{w(x^2)}{8} \\
3M_p &= w \left( \frac{1}{2} - x \right) \left[ \frac{1}{2} + \frac{3}{4} - \frac{x^2}{4} \right] \\
3y &= \frac{w(x)}{8} \left( \frac{3}{4} - x \right) \left( \frac{3}{4} - \frac{x^2}{4} \right) \\
3x^2 &= \frac{1}{8} - x \left( 3x - 2x \right) \\
3x^2 &= \frac{9x^2}{8} - 3 - 8x + 4x^2
\end{align*}$$
\[ W = \frac{8 \times P}{l^2} = \frac{8}{\left(3944 \times 1\right)^2} = 51.43 \text{ MPa} \]

Out of various possible mechanisms, the correct mechanism is the one for which the loading is minimal. Hence, mechanism 2 is the correct mechanism. The critical collapse load will be

\[ \left(\frac{40 \text{ MPa}}{\text{m}^2}\right) \]

Beam is of uniform cross-section. Find the collapse load.

Note:

\[ \frac{A}{B} \]

Two mechanisms need to be checked:

Mechanism 1: PM at A + B
Mechanism 2: PM at A + C
Continuous Beam.

Beam is of uniform cross section, find the collapse load.

In case of continuous beam failure of individual span is considered and the correct mechanism will be the mechanism corresponding to which loading will be minimum.

In this case failure of AB & BC is considered as two mechanisms.

Mechanism 1. Failure of AB

Beam segment AB will fail if plastic hinges are developed one at B and other somewhere between A and B.

\[ \alpha = \frac{w_1 \cdot 444 - w \cdot (444.4)^2}{2} \]

If plastic moment at B and in the span are same, plastic hinge will form at \( 0.414 \cdot l \) from the hinged end A.
continuous flex beam shown above has uniform x sec. find the value of $M_p$. The given loads are collapse load.

For the continuous beam each of the span will be considered separately for it’s failure and $M_p$ required for each of the span will be calculated. If a uniform section is to be provided, we will adopt see corresponding to largest $M_p$.

For failure of AB two hinges are required. It will form at B and other at 0.414 I from hinged end (end A).

$M_p = \frac{6864 \times 4}{3} = 1287 \text{ kNm}$

If instead of collapse load working load is given and $M_p$ required is to be found out, we will have to load the structure with collapse load.

$\text{Collapse load} = \text{load factor } \times \text{ working load}$.
Span BC - BC will fail with x = 5.11 m. Two will form at B & C and one at location of zero弯矩.

\[ \text{Location of zero弯矩} = \frac{17.5}{3.0} = 5.833 \text{ m} \]

\[ \text{Free B.M.D. ordinate} = 175 \times 5.833 - \frac{30 \times 5.833^2}{2} = 510.47 \text{ kN.m} \]

\[ 2M_p = 510.47 \]

\[ M_p = 255.237 \text{ kN.m} \]

For failure of CD hinge will form at C & D below either of point loading.

\[ \text{CD Mechanism 1} \]

\[ \text{Free B.M.D. ordinate} \]

[Diagram of beam with loads and forces]
\[ \frac{2}{2} \cdot m_p = 200 \]
\[ m_p = 200 \]

**Mech. 2**

\[ M_p + \frac{m_p}{4} = 250 \]
\[ \frac{5m_p}{4} = 250 \]
\[ m_p = 200 \]

Hence largest \( m_p \) will be selected, thus \( m_p \) required is 255.178 KNm.

---

8. A continuous beam as shown below is subjected to a collapse load system where each span has a uniform load. If under the action of collapse load system all the spans should collapse, determine the plastic moment required for each span. Assume middle span to be lightest.

\[ M_p \text{ at } C < M_p \text{ at } AB \]

\[ 800 \text{ KN} \]

\[ 160 \text{ KN/m} \]
When two sections meet at a point, plastic hinge forms corresponding to smaller $M_p$

**Span BC**

$$2M_{pe} = \frac{wt^2}{8}$$

$$M_{pe} = \frac{wt^2}{16} = \frac{160 \times 25}{16} = 250 \text{ kNm}$$

**Span AB**

$$M_{pAB} = \frac{M_{pAB} + 250}{2} = 1000 \text{ kNm}$$

$$M_{pAB} = 587.33 \text{ kNm}$$

**Span CD**

30% shear location

$$M_{te} = \frac{160 \times 2 \times 18.75^2}{2} = 382,812.5 \text{ kNm}$$
Seem is uniform find \( M_p \) req. to be provided. Given loading are collapse loading.

Various mechanism are

Mech 1

\[
\frac{N_p + 0.5M}{2} = \frac{200 \times 3}{4}
\]

\[ M_p = 100 \text{ KNm} \]

Mech 2

\[
\frac{3M_p}{2} = \frac{100 \times 4}{4}
\]

\[ M_p = \frac{200}{3} = 66.67 \text{ KNm} \]

Mech 3 combined

\[ 2M_p \theta + 2M_p \theta = 200 \times 1.5 \theta + 100 \times 2 \theta \]

\[ 4M_p \theta = 520 \theta \]

\[ M_p = 125 \text{ KNm} \]

Section will be provided corresponding to \( M_p = 125 \text{ KNm} \)
The collapse of beam will occur only corresponding to x = 3.

To draw collapse, end moment at B needs to be calculated.

Diagram:

[Diagram of a structural beam with labeled points and lines, indicating the collapse points]
Plastic Analysis for Portal Frames.

In the portal frame, various mechanisms considered are:
1) Beam Mechanism  
2) Sway Mechanism  
3) Gable Mechanism  
4) Combined Mechanism  
5) Joint Mechanism.

Beam mechanism in AC  

Beam mechanism in CE

Sway Mechanism.
Mech 1 + Mech 3

Eliminate 2 due to opposite moment at 2

When beam & sway mechanism are considered combining as in this case if we consider only 2 & 3, no of hinges will be five but max. no. of hinge that can be formed is 4 only hence one of the hinge needs to be eliminated. The common hinge b/w beam & sway mechanism are 2 & 3.

At 2, the moment from beam and sway mechanism are opposite in nature but they are of same nature at 3 hence in combined mechanism hinge at 2 will be eliminated.
Combined Mechanism

\((1+3)\)

\[\mathbf{\phi} = L_2 \phi\]

Find collapse load \(W\) and draw collapse BMD for the portal frame shown above.

\(R = 3\)

max. no. of plastic hinge = 4

Beam mechanism BC

\(\phi(\mathbf{\phi})\)
At a full force due to 1.5 \text{mp} \text{ when all forces at joint due to smaller mp.}

\[ 3 \Phi = 4 \Phi \]

\[-2 \text{mp} \theta - 2 \text{mp} \theta - 2 \text{mp} \phi + 1.5 \text{mp} \phi + w \cdot 3 \theta = 0 \]

\[ +4 \text{mp} \theta + 3.5 \text{mp} \cdot \frac{3}{4} \theta = 3 \text{w} \theta \]

\[ \theta = \frac{5.3}{2.4} \text{mp} = 2.108 \text{mp} \]

\[
\text{Sway mechanism.}
\]

\[
\frac{w}{2} \rightarrow 1.5 \text{mp} \]

\[ \phi \]

\[ \frac{w}{2} \]

\[ \text{Combined mechanism} \]

\[ 6 \theta = 4 \phi \] (i)

\[ 3 \theta = 4 \alpha \] (ii)

\[-2 \text{mp} \theta - 2 \text{mp} \theta - 2 \text{mp} \phi + 1.5 \text{mp} \phi - 3 \text{mp} \phi = -w \cdot \theta - \frac{w \cdot 6 \theta}{2} \]

\[-4 \text{mp} \theta - 3.5 \times 3 \text{mp} \theta - 3 \times 1.5 \text{mp} \theta \]

\[
\theta = 1.85 \text{mp}
\]
Out of various possible mechanisms correct mech. is one for which collapse loading is min.
Hence collapse load is 1.854 MP.

Collapsed B.M.O.

\[ \frac{V}{2} \rightarrow \sum_{m} (M_{A} + \frac{H_{A}w}{2}) = 1.5 \text{ MP} \]

\[ H_{A} \rightarrow \sum_{m} (M_{B} + \frac{H_{B}w}{2}) = 1.5 \text{ MP} \]

\[ 2 \text{ MP} \]

\[ H_{A} + H_{B} + \frac{W}{2} = 0 \] (i)

\[ \sum_{m} M = 0 \text{ for } CD \text{ portion} \]

\[ 1.5 \text{ MP} + 1.5 \text{ MP} + H_{B}xq = 0 \]

\[ H_{B} = \frac{-3}{4} \text{ MP} = -0.75 \text{ MP} \]

\[ H_{A} = -H_{B} - \frac{W}{2} = -1.77 \text{ MP} \]

\[ \cdot 73 \text{ MP} \]

\[ 1.5 \text{ MP} \]

\[ 2 \text{ MP} \]

\[ U \]
PLATE GIRDERS

Plate girders (beams made using plates) adopted when span and loading becomes large. For span upto 10 m & seen, plates may be used but for larger span 15 to 30 m plate girders are used.

As the plate girders are deeper chances of web buckling is more hence web has to be supported by using stiffeners. Stiffeners actually increase the moment of inertia.

\[ a, b \] are unsupported length.

Horizontal stiffeners

Vertical stiffeners/branched stiffeners

Load bearing stiffeners
Design of Web Plate

Web primarily resist shear hence web is designed for shear.

For unstiffened web permissible web av shear stress = 0.4 fy

\[ \sigma = \frac{f}{b_w c} \]

For stiffened, \( c = 2b \)

Economical depth is given by

\[ d = 1.1 \sqrt{\frac{m}{\delta_{bt} t_w}} \]

Thickness will be assumed and \( d \) will be calculated from economical depth formula and it will be ensured that with this dimension \( \frac{V_{max}}{d^2 b_w} \) \( \leq \) per av shear stress.

Spacing of vertical stiffeners will be chosen such that the calculated average shear stress is less than equal to per av shear stress.

Design of Flange Plate

\[ I = 2 \left( A_f \left( \frac{d}{2} \right)^2 + \frac{b_u d^3}{12} \right) + \left( A_f + A_w \right) \left( \frac{d^2}{2} \right) \]
Flange plate is designed to resist bending moment.

\[ M_{OR} = \sigma_{tt} \times Z \]

\[ N_{rms} = \sigma_{bt} \times \frac{b}{d/2} \]

\[ = \sigma_{bt} \left[ \left( A_f + \frac{A_o}{8} \right) \right] d \]

\[ \text{Effective flange area on compression side} \]

\[ = \frac{A_f + A_o}{8} \frac{d^2}{2} \]

On tension side, reduction in flange area will occur due to formation of holes. Hence, the area of flange plate has to increase. The effective flange area for tension side is taken as \( A_f + \frac{A_o}{8} \).

\[ \left[ M_{max} = \sigma_{tt} \left( A_f + \frac{A_o}{8} \right) d \right] \]

Area of flange plate calculated using this formula will be more.

For design purpose, flange area is calculated on tension side and same is adopted for compression side.
Check for bending

\[
\frac{M_{max} \cdot y_{max}}{I_{flange}} \leq \sigma_{bc}
\]

\[
\frac{M_{max} \cdot y_{max} \times (G.A. \text{ of tension flange})}{I_{flange} \times \left(\text{Net area of tension flange}\right)} \leq \sigma_{bc}
\]

Tension flange will comprise of flange plate, flange angles and area of web plate included by flange angle

Design of stiffner

1) \( \frac{d}{t_w} < 85 \) No stiffner required (Heber or vertical stiffner)

2) \( 85 \leq \frac{d}{t_w} \leq 200 \) Only vertical stiffener is required

3) \( 200 < \frac{d}{t_w} \leq 250 \) Apart from vertical stiffener one horizontal stiffener is provided at a distance from compression flange equal to \( \frac{2}{5} \) th of the distance of compression flange from N.A.

4) \( \frac{d}{t_w} \in [250 - 400] \) One more hor. stiffner is provided at N.A.

\( 250 < \frac{d}{t_w} \leq 400 \)
for stiffened web min thickness of web \( t_w = \frac{d}{400} \)

for unstiffened web \( t_w \) min = \( \frac{d}{85} \)

- Design of vertical stiffeners

Spacing is selected such that permissible shear stress is greater than \( \frac{V}{d t_w} \).

The max. and min. spacing of vertical stiffeners are 1.5 \( d \), or 0.33 \( d \), respectively.

To decide about the type of stiffener to be provided we work on actual length of web however to design the vertical stiffener i.e. to decide on the cross-section of stiffener required, we work on \( d_1 \) (actual unsupported length).

\[ d_2 = \text{twice the clear distance from N14 of the beam to the compression flange} \]

\[ d_2 = 2x \]

Larger or smaller clear dimension should be maintained below:

270 bo & 180 bo
Vertical stiffener should be provided on one angle two sides of web. If it is provided on one side it should be staggered.

\[ \text{Vertical stiffener} \]


\[ \frac{\text{Moment of Inertia of angle}}{1.5 \times d \times l_w} \leq \frac{c^2}{e^2} \]

The moment of inertia of angle should not be less than

Connection Design

Connection should be designed for S.F.

\[ S_f = \frac{12.5 \times t_w^2}{h} \text{ N/mm} \]

Where \( h \) is the outstand of stiffener from the web

\[ \frac{f_u}{12.5 \times b_o^2} = \text{Pitch} \]

\( t_s \neq 16 \times t \) for angle where \( t \) is the thickness of member.
In case of plate end change \( f = \frac{1}{l c} \)

- \( l c \) thickness of plate.

Vertical stiffener is primary supposed to resist shear buckling.

**Horizontal stiffener**

1. \( 200 < \frac{d_2}{l c} < 250 \)
2. \( 250 < \frac{d_2}{l c} < 400 \)

\( I \neq \frac{1}{4} c \) \( c \) \( \frac{1}{2} d_2 \) \( d_2 \) \( c \) \( \frac{1}{2} d_2 \)

**Connection of horizontal stiffener will be done in same way as vertical stiffener.**

Horizontal stiffener primarily safeguards against buckling due to bending compression.

**Design of load bearing stiffener**

- \( A_1 \) - Working area
- \( t \) - Thickness of flange angle
- \( t_w \) - Thickness of web
- \( t_l \) - Load bearing stiffener

**Load bearing stiffener**
1. Outstanding leg of the stiffener is assumed to provide the necessary bearing area hence bearing area required is found out

\[ \frac{1}{\delta_{sw}} = \text{bearing area req.} \]

\( \delta_{sw} \rightarrow \text{perm. bearing stress} = 0.75 \text{fy} \)

2. Unequal angle is chosen in such a way that the overhang portion provides the necessary bearing area. Smaller leg will be connected to the web larger leg will be overhang.

3. Once the angle is chosen overall buckling is to be checked.

4. To check for overall buckling the stiffener is assumed to be a column. The permissible load capacity of that column should be greater than applied load. To find out \( \lambda \) is required and

\[ \lambda = \frac{l_{eff}}{\delta_{min}} \]

Where \( l_{eff} \) is taken as 0.7 \( l \)

\( l \rightarrow \text{overall depth/length of stiffener} \)

The section resisting compression is assumed to be consisting of a pair of stiffeners together with a length of web on either side of centre line of stiffener, where available, equal to 20 times the web thickness.
A: shaded area
= 40 \text{ box} \times h_t + 2A

Load carrying capacity
= \sigma_{tc} \times A > P

Web Splices

As Cram

In Fig (i)
In Fig. (i) when splice plate will be assumed to carry only shear at the splice location and moment equal to $M_u$.

where
$$M_u = \frac{M_{lo}}{I}\text{.}$$

where $M_{lo}$ B.m. at splicing section
$I_u$ - m.o.i. of web plate
$I$ - geo.m.o.i of original seen about bending axis.

$$\frac{V}{2d^2t} \leq 0.4fy$$

$$M_{OR} = \frac{\sigma_{bt} \cdot 2t \cdot d^2}{6} \
\geq M_u$$

Connection will be designed as eccentric connection

for duct s.f. $\gamma$ 

$$f\text{ Torqional moment } = M_w$$

In Fig. (ii) we provide two sets of plate

1. Shear splice - assumed to resist only shear
2. Moment splice - shear at splicing seen
   - assumed to resist moment $M_w$
Moment splice

\[ F \alpha = M_0 \]

\[ F = \frac{M_0}{\alpha} \]

\[ 2 \left( b - d' \right) \times t_f \times \sigma_{at} \geq F \]

\[ N = \frac{F}{R_n} \]

This design is for tension and some area is provided for compression.

Shear splice

\[ \frac{V}{2d_0 \times t} \leq 0.4 f_y \]

\[ A = \frac{V}{\rho_v} \]
Industrial Building

- King post truss
  - Span < 6 m
- Queen post truss
  - 9 m

- Fink truss
- Howe truss
  - 12 - 18 m
- Howe truss (Flat)
  - Up to 24 m

- Compound fink
  - 12 - 18 m

- Saw tooth
- Glay
  - Glay for sun light

- Height of truss
  \[ \text{Height of truss} = \frac{H}{L} = \frac{2(H)}{L} \]

- Slope of truss
  \[ \text{Slope} = \frac{H}{L} \]

- Pitch of truss
  \[ \text{Pitch} = \frac{H}{L} \]

- Slope = 2 x Pitch
Deeper trusses have smaller deflection as compared to shallow trusses.

Member forces are larger in shallow truss as compared to deep truss.

For smaller span height of truss is normally taken as $\frac{1}{6}$th of the span.

$$\frac{h}{L} = \frac{1}{6}$$

Slope $= \frac{1}{3}$

For larger span depth may be taken as $\frac{1}{10}$th of the span for Galvanized iron.

If GI sheet is used as roof covering pitch of truss may be taken as 1 in 6 for AC (asbestos cement) sheet roof covering pitch may be taken as 1 in 12.

Lower pitch is advantageous for wind pressure.

Spacing of truss.
Normally spacing is taken \( \frac{1}{4} \) of span. Span upto 15m.

\( \frac{1}{5} \) of span, for span > 15m.

For economy in roof, the cost of truss is divided by area.

\[
\text{Cost of truss} = \frac{\text{Cost of purlin}}{\text{Area}} \times 2 + \text{Cost of roof covering}
\]

Area = Area of floor (or plan area of building).

# Purlins

The purpose of purlin is to support the roof sheet.

- Sag rod
- Tension member
- Purlin are channel, angle or I-section.
Max spacing b/w purlin is 1.4 m

Purlins are subjected to bi-axial loading.

Hence design of it should be done as bi-axial bending.

However if slope of roof truss is $< 30^\circ$

We may avoid design purlin by bi-axial bending.

In this case ($< 30^\circ$) angle purlins can be used and it will be designed as uniaxial bending condition.

In this case B. L. I. & W. L. are assumed to be acting 1 to the roof coverage.

\[ \text{Purlin} \]

\[ M_{\text{max}} = \frac{W \times L}{10} \]
- \( l \) = span of purlin

- \( \bar{w} \) = spacing b/w truss

- \( w \) = Total load resisted by purlin

\[ Z_{xx} = \frac{wL/10}{\sigma_{fc}} \]

\[ Z_{xx} \geq \frac{wL/10}{\sigma_{fc}} \quad \text{choose angle such that} \]

\[ x \geq \frac{L}{60}. \]

\[
\begin{align*}
\text{Angle chosen should be such that} \\
\text{width of angle } \theta \text{ to roof covering} & \geq \frac{L}{45} \quad \text{Recommended only for angle purlin} \\
\text{Sag and resist tangential component of } \frac{l}{60} \text{ due to sheet (roof covering) or purlin itself.}
\end{align*}
\]

Try to make overlap connection at the location of purlin
For slope upto 1:3

- For flatter slope, overlap length can be reduced.

\[ \frac{1}{2} \text{ Corrugation length lap} \]

Side lap for A.C. sheet is \( \frac{1}{2} \) corrugation length lap.

For G.I. sheet, it is full corrugation length lap.

Note: When two sheets overlap at location, two sheets are cut at corners. This process is called mitring.
Most of the time in a year, the sun moves from east to west through south hence direct sunlight will come in the building from south which will cause shadow formation. To avoid it, windows should be provided on the north side so that diffused light come inside the building which does not cause any shadow formation.